

## Trans-Dimensional multimode surface wave inversion of DAS data at the CaMI-FRS

*Luping Qu, Jan Dettmer, and Kris Innanen*

### Summary

Inversion of surface-wave seismic data to aid in the characterization of the near-surface is a rich and well-explored field. However, several new areas of technology development, both algorithmic and acquisition-based, have the potential to lead to significant improvements in the accuracy and availability of near-surface models. To this end, in this study we implement a trans-dimensional multimode surface wave dispersion inversion, which incorporates the multimodal phase-velocity dispersion curves of the Rayleigh wave, and we apply it to Distributed Acoustic Sensing (DAS) data acquired with a trenched fiber-optic cable set at the Containment and Monitoring Institute-Field Research Station. The combination of multiple modes, in a stochastic sense, is explained in detail in this work. For the cautious treatments of real data, a comprehensive spectral analysis and error estimations on DAS data are carried out. Besides, a new mode separation method, combining Fourier transform and dispersion compensation, permitting clear picking of dispersion curves, is adopted for the DAS data. With the dense spatial sampling, DAS data show better data quality for dispersion curves picking and inversion, which correspondingly provides more underground information. In addition, better-resolved models are obtained with the incorporation of higher modes both in synthetic and field tests. Through the comparison, improvements in model recovery of multimode inversion are clearly shown, and the great potential of DAS data for subsurface characterization is revealed.

### Theory

#### a). Dispersion Compensation

For the real surface wave data, an effective mode separation method should be adopted to conduct mode separation. Here, a method combining Fourier transform and dispersion compensation method is used (Xu et al., 2012). First, the data are transformed into frequency domain, and an initial mode separation is conducted. Then the dispersion compensation is used for a finer separation. Theoretically, the multimodal Rayleigh wave data can be treated as the response of a broadband source  $F(\omega)$ . The dispersive phenomenon can be treated as a phase shifting of a constant velocity propagation. Therefore, the multimode surface wave so  $S(\omega)$  in frequency domain at a certain offset  $x$  could be expressed as

$$S(\omega) = \sum_{i=1}^N Amp_i T_i(\omega) F(\omega), \quad (1)$$

where  $T_i(\omega) = \exp(-jk_i(\omega)x)$ ,  $k_i(\omega)$  is the dispersion phase shifting from original signal.  $Amp_i$  is the amplitude for each mode. The dispersion could be reversed by dispersion compensation, which is done through a reversed phase shifting. The process can be complemented by multiplying  $T_i^{-1}(\omega) = \exp(jk_i(\omega)x)$ , which gives

$$S^*(\omega) = T_i^{-1}(\omega) S(\omega) = Amp_i F(\omega) + T_i^{-1}(\omega) \sum_{j=1, j \neq i}^N Amp_j T_j(\omega) F(\omega). \quad (2)$$

$S^*(\omega)$  is the frequency spectrum after dispersion compensation. Different velocities  $k_i(\omega)$  can be used to implement the dispersion compensation. Then, noises and other unwanted signals with different velocities can be over-shifted or under-shifted, and filtered with a cosine taper

filtering, when the dispersive velocity of a certain mode is picked. Different modes can be extracted in the same way.

b). Likelihood formulation for multimode inversion

In linear surface wave inversion, multimode inversion misfit is the summation of the 2 norm residuals. While in Bayesian inversion, the multimode inversion should not be summation, as the misfit is the likelihood in essence.

In statistics, the probability of two independent events occurring together equals to the multiplication of their likelihoods. As there is no explicit relation between the fundamental mode and the higher modes, we will treat them as independent. Therefore, the likelihood of the model that meets both the fundamental and higher modes of phase velocity dispersion curves is the product of the model likelihoods which fit all those dispersion curves (Li et al., 2012) expressed as

$$L(\mathbf{m}) = \prod_{i=1}^S \frac{1}{\sqrt{(2\pi)^{N_i} |\mathbf{C}_{di}|}} \exp\left(-\frac{1}{2} \mathbf{r}_i^T \mathbf{C}_{di}^{-1} \mathbf{r}_i\right). \quad (3)$$

$i$  could be the mode index for phase velocities which are used in the inversion process with a total number of  $S$ .  $\mathbf{r}_i$  are the data residuals between the measured data and the synthetic data.  $\mathbf{C}_{di}$  is the data covariance matrix for a specific dispersion curve and  $\mathbf{m}$  represents all the model parameters.

**Results**

A three-layered simulated model over a half-space is used to test the multimode trans-dimensional inversion method (Dettmer et al., 2012). 1 percent magnitude scaled Gaussian distributed random noise is added to these test data.

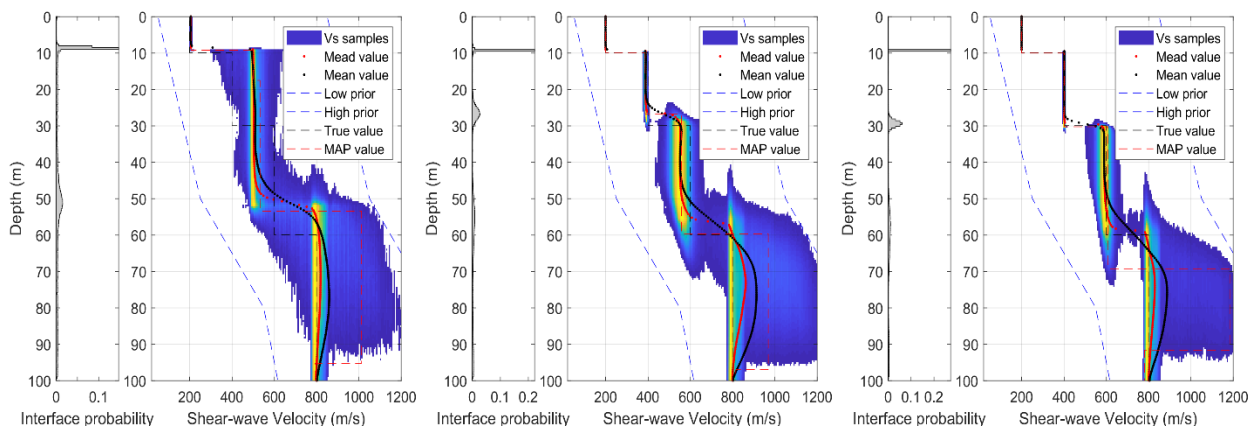


Figure1. The inversion result of using the fundamental mode, first two modes, and first three modes (black dashed line is the true model).

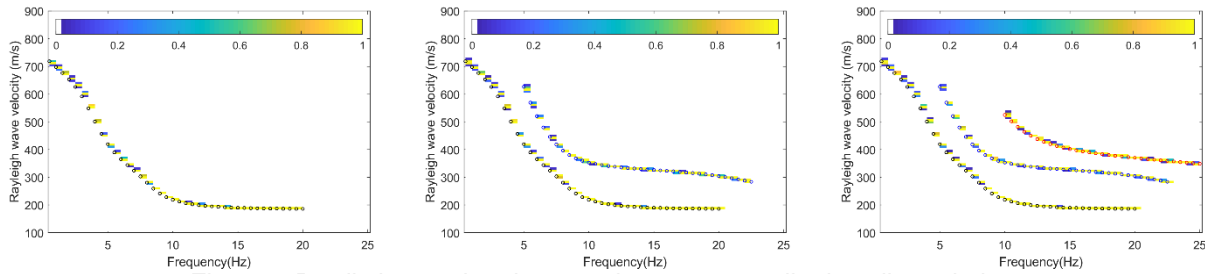


Figure2. Predictions using the samples correspondingly collected above.

From the comparison of inversion results incorporating different modes, we can find multimode inversion result indicates well-defined near surface structure and substantially narrower uncertainty at all depth.

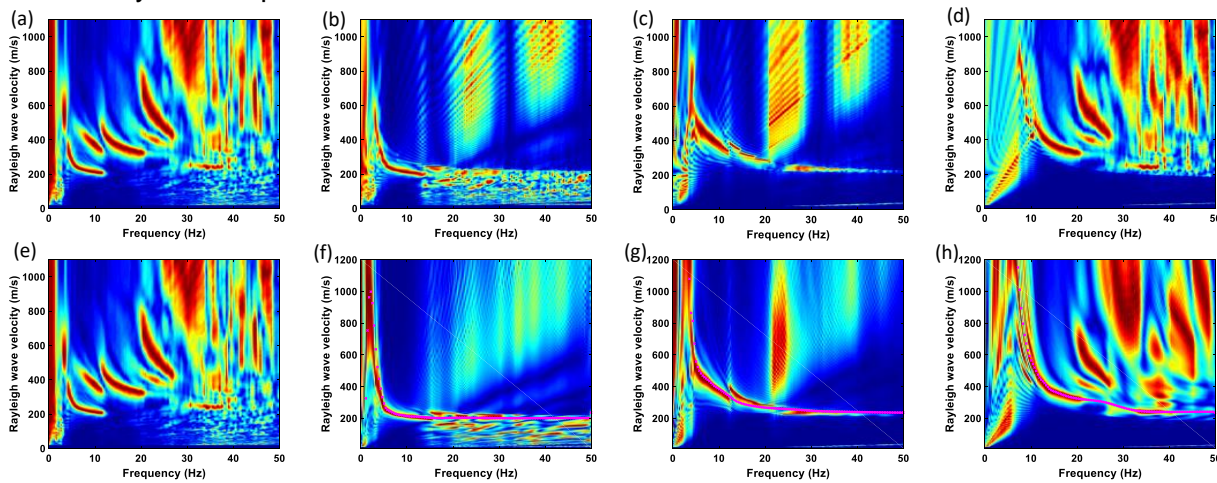


Figure3. Mode separation for DAS data. (a) is the original spectrum. (b), (c), and (d) are fundamental mode, first higher mode, and second higher mode after f-k filtering. (f), (g), and (h) are fundamental mode, first higher mode, and second higher mode after mode separation.

Then, the method is tested on real DAS data. The initial  $V_s$  inversion results without error estimations (Dettmer et al., 2007) are shown in Figure4. With a more cautious error estimation, the  $V_s$  inversion results are shown in Figure5.

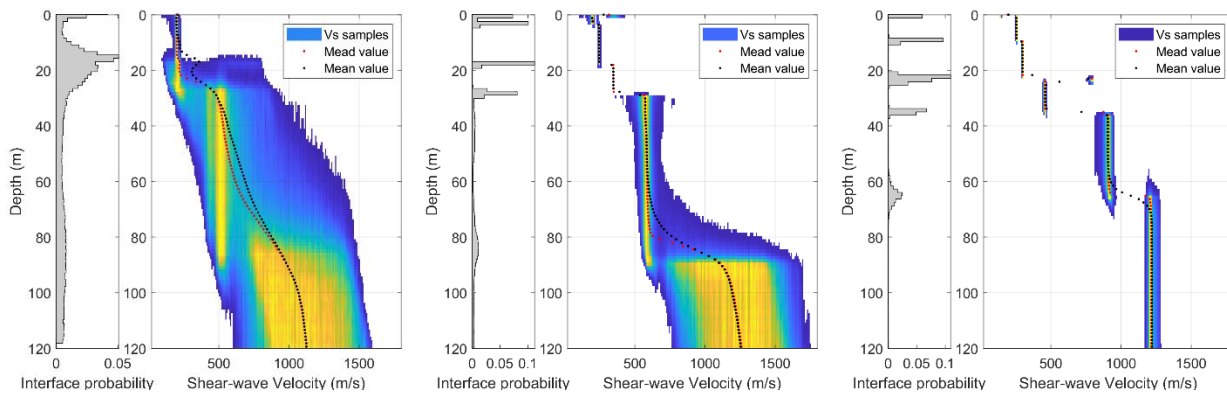


Figure4. The 1<sup>st</sup> inversion results of DAS data at CaMI-FRS

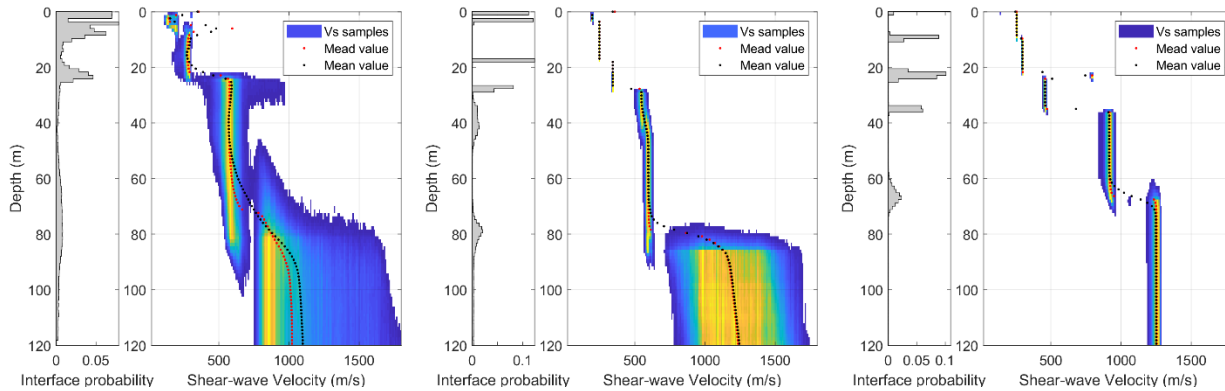


Figure5. The inversion results of DAS data after error estimation.

In addition, the inversion result of DAS data shows great consistency with the geology information.

## Conclusions

This study focused on utilizing surface DAS data for multimode surface wave dispersion inversion. Surface DAS data provide densely sampled Rayleigh wave with no interference of other waves. Considering computational cost and geology background, an efficient and probabilistic inversion named trans-Dimensional inversion is used for quantitatively characterizing the distribution of shear-wave velocity, layer thickness and layer number in shallow site. Besides, a rigorous data error estimation method is adopted to include correlated errors and non-stationary errors into the inversion process. For more efficient convergence, parallel tempering which allow dynamic state exchange between different Markov Chains and principal component rotation for better convergence are used to optimize the inversion algorithm. The inversion applied to simulated models with random data error and correlated data errors. The Vs marginal profile conforms with the true model and demonstrates reasonable uncertainty distribution. The data prediction ensemble generated by PPD samples could fit the data well. Then we applied it to multimode DAS data, and we found the result is consistent with other results. Better characterization of the shallow site is obtained

## Acknowledgements

We thank the sponsors of CREWES for continued support. This work was funded by CREWES industrial sponsors, NSERC(Natural Science and Engineering Research Council of Canada) through the grant CRDPJ 461179-13. Luping was partially supported by a scholarship from SEG. Thanks to the Containment and Monitoring Institute and all the field staff for DAS data collection.

## References

- Dettmer, J., Dosso, S. E., and Holland, C. W., 2007, Uncertainty estimation in seismo-acoustic reflection travel time inversion: *The Journal of the Acoustical Society of America*, 122, No. 1, 161–176.
- Dettmer, J., Dosso, S. E., and Holland, C. W., 2010, Trans-dimensional geoacoustic inversion: *The Journal of the Acoustical Society of America*, 128, No. 6, 3393–3405.
- Xu, K., Ta, D., Moilanen, P., and Wang, W., 2012, Mode separation of lamb waves based on dispersion compensation method: *The Journal of the Acoustical Society of America*, 131, No. 4, 2714–2722.



Li, C., Dosso, S. E., Dong, H., Yu, D., and Liu, L., 2012, Bayesian inversion of multimode interface-wave dispersion from ambient noise: IEEE Journal of Oceanic Engineering, 37, No. 3, 407–416.