

# A deep learning formulation of elastic FWI with numerical and parameterization analysis

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## Summary

In this study, a theory-guided recurrent neural network (RNN) is designed to achieve isotropic elastic full waveform inversion. Each of the RNN cell is designed according to the isotropic elastic wave equation. The final output of this network are the wavefields and the synthetic shotrecords at our last computational time. The velocity models would be used as the trainable parameters for this network. Based on the Automatic Differential method, the gradient, which acts to update the elastic models can be constructed by inspection and use of the computational graph. In order to mitigate the cross-talk between eFWI, we use different parameters to release this issue.

## Introduction

The project reported in 2018 (and published this year) which formalized the approach for us was the work of Sun et al. (2019), in which a recursive neural network was set up to simulate the propagation of a seismic wave through a general acoustic medium. The network is set up in such a way that the trainable weights correspond to the medium unknowns (i.e., wave velocity model), and the non-trainable weights correspond to the mathematical rules (differencing etc.) enforcing wave propagation. The output layer was the field projected onto a measurement surface. Sun et al. (2019) discovered that training such a network with a single data set is very close to carrying out full waveform inversion.

In this study, we extend their idea to perform the 2D isotropic elastic full waveform inversion, by using the recurrent neural network (RNN). In order to tackle with the cross-talk problem, we also use different parameterization to mitigate this problem. Three types of parameterizations are used in this test. The velocity parameterization (V-D), the modules parameterization (M-D), and the stiffness matrix parameterization (S-D). The numerical test shows that the 2D isotropic elastic full waveform inversion based on this recurrent neural network can give promising inversion results and the different parameterization can help you to mitigate the cross-talk problem and finally improve the inversion results.

## Theory and method

### 1. Forward modeling

Equation (1) shows the equation of the isotropic elastic wave equation.  $v_x$  and  $v_z$  are the elastic velocity fields in x and z direction.  $\sigma_{xx}$ ,  $\sigma_{zz}$  and  $\sigma_{xz}$  are the stress tensors.  $\rho$  is the density of the model.  $\lambda$  and  $\mu$  are the first and the second Lamé parameter for the elastic media.

$$\begin{aligned}
 \frac{\partial v_x}{\partial t} &= \frac{1}{\rho} \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} \right) \\
 \frac{\partial v_z}{\partial t} &= \frac{1}{\rho} \left( \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} \right) \\
 \frac{\partial \sigma_{xx}}{\partial t} &= (\lambda + 2\mu) \frac{\partial v_x}{\partial x} + \lambda \frac{\partial v_z}{\partial z} \\
 \frac{\partial \sigma_{zz}}{\partial t} &= (\lambda + 2\mu) \frac{\partial v_z}{\partial z} + \lambda \frac{\partial v_x}{\partial x} \\
 \left( \frac{\partial \sigma_{xz}}{\partial t} \right) &= \mu \left( \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right)
 \end{aligned} \tag{1}$$

To simulate the wave propagation in the underground world, a PML (Perfect Matching Layer) damping coefficient is used to absorb energy when the waves hit the boundary of the model.

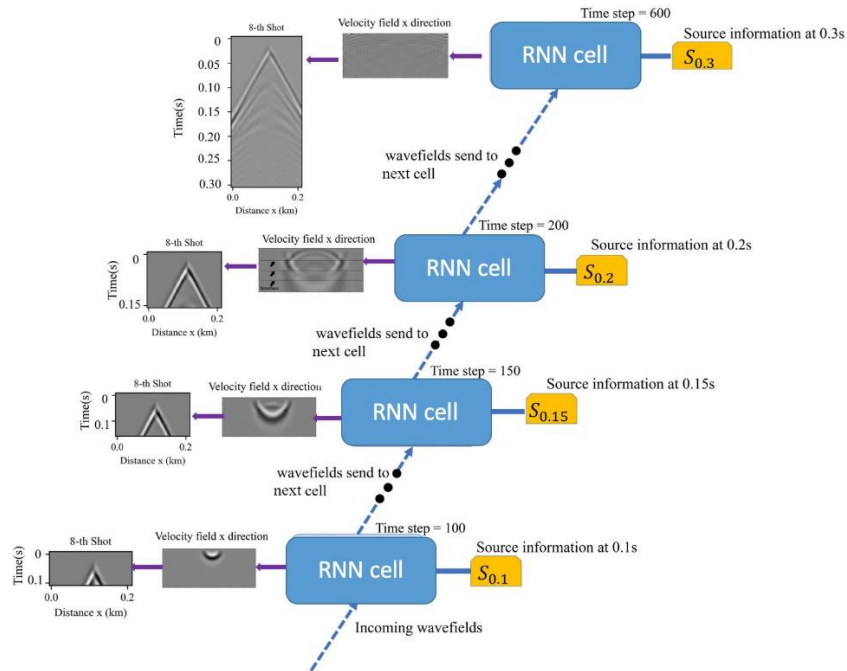


Figure 1. How elastic RNN generate synthetic data

Figure 1 shows how elastic RNN generates seismic records. Every RNN cell is designed according to the isotropic elastic wave equation, which is equation (1). In this study, we use the staggered grid finite difference method to simulate synthetic data, and we use image convolution operation in machine learning to calculate space partial derivative.

## 2. Automatic differential

Gradient calculation is one of the most important steps for FWI. The gradients in this study are calculated by the Automatic Differential engine in the machine learning library, Pytorch. The mathematical principle under this Automatic Differential engine is the Chain's Rule. During forward

propagation, RNN would generate synthetic shotrecords at each time step, at the same time, a Dynamic Computational Graph (DCG) would be built to record the mathematical operation between each variable, for instance, how the stress field is calculated according to the stress field at the last time step and the partial derivative of the velocity field. Then, we can start the backpropagation method. According to the Dynamic Computational Graph saved in memory, residual between observe data and the synthetic data would be calculated and backpropagated to the trainable parameters. After calculating the gradient, we need to use an optimization method to calculate the direction and update the trainable parameters

## Numerical tests

### 1. Synthetic test

Figure 2(b), (e), (h) are the initial models we use for  $V_p$ ,  $V_s$  and  $\rho$  model. Figure 2 (c) is the inversion result of the  $V_p$  model. Figure 2 (f) is the inversion result of the  $V_p$  model. Figure 2 (i) is the inversion result of the density  $\rho$  model. Compared with the true model, the layers of the subsurface have been correctly updated, which means that this elastic FWI deep learning method based on recurrent neural network could provide us with promising inversion results. The matching between the inversion results and the true models are very satisfactory.

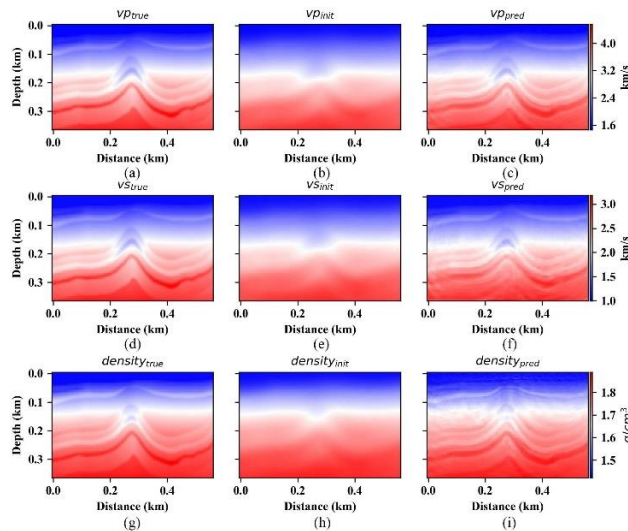


Figure 2 (a), (b), (c) True, initial and inversion result for  $V_p$  model respectively, (d), (e), (f) True, initial and inversion result for  $V_p$  model respectively,(g), (h), (i) True, initial and inversion result for  $\rho$  model respectively.

### 2. Different parameterization

Estimating multiparameter models is an essential step for lithologic characterization and reservoir monitoring. However, if we intend to update several parameters simultaneously, the update of one parameter would influence other parameters, and this is the trade-off problem or also referred to the cross-talk problem. In this test we use different parameterization to mitigate this problem.

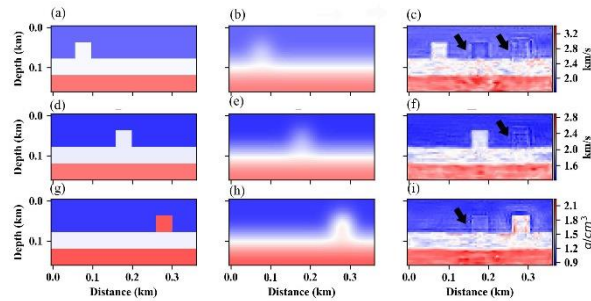


Figure 3. Velocity parameterization: (a)-(c) True, initial and inversion result for  $V_p$  model respectively, (d)-(f) Inversion result for  $V_s$  model (g)-(i) Inversion for  $\rho$  model.

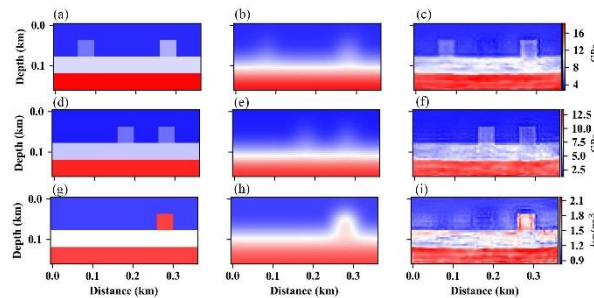


Figure 4. Stiffness matrix parameterization: (a)-(c) True, initial and inversion result for  $C_{11}$  model respectively, (d)-(f) Inversion for  $C_{44}$  model, (g)-(i) inversion for  $\rho$  model.

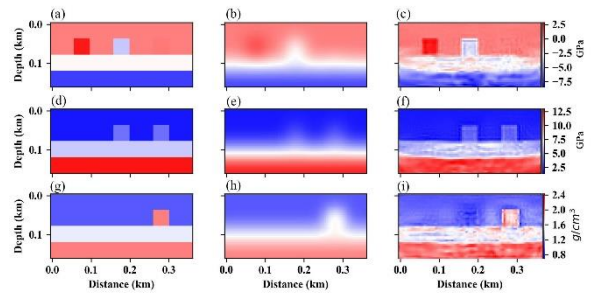


Figure 5. Modulus parameterization: (a), (b), (c) True, initial and inversion result for  $\lambda$  model respectively, (d)-(f) inversion for  $\mu$  model, (g)-(i) inversion for  $\rho$ .

Figure 3 shows the inversion of another model by using the V-D parameterization, and figure 4 demonstrates the inversion results of this model by using the S-D parameterization. Figure 5 shows the inversion by using the Modulus parameterization. We can see that all the three parameterizations can give the correct inversion results, however the S-D and M-D parameterization suffers less from the cross-talk problem. The inversion results for density are also better by using the S-D and V-D parameterizations.

## Conclusions

In this study, we use the recurrent neural network to perform 2D isotropic elastic FWI, which forms a theory-based machine learning method. In this network, each of the RNN cell is designed

according to the isotropic elastic wave equation. The gradient is given automatically by using the Automatic Differential engine imbedded in the machine learning library we use, the Pytorch. Numerical test tells that the eFWI based on this RNN can give promising inversion results and the using of different parameterization can help to mitigate cross talk problem and improve inversion results.

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