



## Tunneling: an approach for including rock-physics constraints in full-waveform inversion

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### Summary

Highly accurate models of elastic subsurface properties can be obtained using full-waveform inversion (FWI) of seismic data. However, the large computational burden of wavefield modeling typically means that local optimization methods must be used in FWI. These methods are poorly suited for cases where prior information suggests clustering in elastic-property space. For instance, conventional FWI struggles to incorporate well-log information suggesting the presence of clusters of elastically dissimilar rocks. We propose an FWI optimization approach that uses non-local prior information in conjunction with strictly local data features to allow for clustering prior information to be used effectively.

### Background

In full waveform inversion, subsurface properties are estimated by finding the subsurface model which produces synthetic seismic data most similar to the measured data. Due to the computational expense of seismic wavefield modeling, the model producing the best match to measured data can generally only be approximated, which is done through minimization of a misfit function, measuring the discrepancy between the modeled and observed seismic data. This optimization procedure is usually solved through local methods, which improve model estimates based on the derivatives of the misfit function, rather than searching all of model space.

The FWI problem can be made considerably simpler when non-seismic sources of information are used to supplement the data-fit term in the misfit function. Including prior information about reasonable spatial structures, bounds on parameter values, or prior best estimates, for instance, have been shown to significantly improve the accuracy and convergence rate of FWI (e.g. Asnashaari et al., 2013 Aghamiry et al., 2019). A significant source of prior information in most seismic settings comes from knowledge of rock-physics information, whether from well-logs in the region, or from geological and petrophysical understanding of the region. Rock physics information can establish restrictions on which combinations of seismic or rock properties are plausible for an inversion result. This type of information can be leveraged very effectively in FWI (e.g. Zhang et al. 2018; Aragao and Sava 2020).

If rock physics information suggests that clusters of rock properties are present, however, it can present a challenging problem for FWI techniques. In this case, an ideal FWI misfit function should have small values only where allowable clusters of rock properties exist, but this can lead to local minima, which severely harm FWI results. As clustering of rock properties is not uncommon, we present here an approach for an efficient non-local optimization in FWI to allow for effective treatment of clustering rock physics priors.

### Theory / Method

To allow for effective treatment of clustering prior information in FWI, we investigate a modification of the conventional approach. We propose augmenting the local model update that occurs at each iteration of a conventional inversion with a potential second update, that we refer to as a “tunneling step”. The tunneling step is designed to allow model elements poorly suited to their current cluster to update to a new cluster, based on the local character of the seismic data fit, and the global character of the prior fit. Because only the prior fit is considered non-locally, the computational expense of this additional step is small.

For a model element to be updated to a new cluster in the tunneling step, we require that two conditions be satisfied: a potential condition and a momentum condition. The potential condition is a requirement that only model elements that do not fit well with the prior information in their current locations be allowed to tunnel:

$$\Phi_R(m) > \Phi_{\text{thresh}}$$

where  $\Phi_R$  is the prior-fit penalty term associated with current model  $m$ , and  $\Phi_{\text{thresh}}$  is a threshold value defining the minimum penalty allowed before a model element can tunnel.

The momentum condition is a requirement that elements not be tunneled to a cluster unless the data-fit term supports an update in that direction. We define the momentum condition for cluster number  $n$  as

$$\Delta m \bullet \hat{b}_n > p_{\text{thresh}}$$

where  $\Delta m$  is the ‘momentum’, a unit vector in the direction from a model element’s initial location in a cluster to its current location,  $\hat{b}_n$  is the unit vector from the current model space location to the center of the  $n$ th cluster, and  $p_{\text{thresh}}$  is a threshold value, defining the minimum dot product between the two vectors that is acceptable. If this inequality is satisfied for a model element for more than one cluster  $n$ , only the cluster with the highest momentum  $\Delta m \bullet \hat{b}_n$  is considered to satisfy the momentum condition.

If the potential condition is satisfied, it suggests that the data-fit term is pushing strongly against the prior-fit term in the local cluster and may not belong in that cluster. If the momentum condition is satisfied, it suggests that the moving the model element in the direction of another cluster may reduce the data-misfit term. When both conditions are satisfied, a model element is moved from its current parameter-space location to the maximum likelihood location in the cluster satisfying the momentum condition.

## Numerical Results

To investigate the behaviour of the tunneling approach, we perform a series of synthetic tests. We use the model in figure 1 (top left) to generate the data used in these tests. Each inversion used a constant initial model with  $v_P$  and density equal to the top layer of the true model. Sources and receivers at the top of the model, with frequencies from 1-20 Hz were used. Frequency-domain inversion was performed, with 10 frequency bands considered. Two parameter  $v_P$ -density inversion was used.

To assess the effectiveness of the tunneling approach, we compare three inversion results: an unregularized inversion, which makes use of data-fit information only, a regularized inversion,

which also considers prior information on parameter relations, but uses conventional FWI optimization, and a tunneling inversion, which uses the same information as the regularized inversion, but makes use of the tunneling approach.

We consider a case in which four regional rock-type trends have been identified and describe the  $v_P$ -density relations shown in figure 2, where the dark grey represents one standard deviation, and the light grey represents two standard deviations from the cluster means. The inversions that use prior information include misfit terms representing these clusters.

Figure 1 shows the results for each inversion strategy. Notably, both the unregularized (bottom left) and tunneling (bottom right) results outperform the conventional regularized inversion (top right), illustrating the challenge of treating clustering prior information in a conventional approach. The tunneling inversion approach outperforms the unregularized inversion in several ways, including better resolution of layers and more accurate parameter estimates.

The distribution of the model elements in  $v_P$ -density space is shown in Figure 2 for the first six of the ten total frequency bands considered. After the first band, each inversion has model elements primarily near the starting values. At later frequency bands, significant differences between the approaches become evident. The unregularized inversion quickly spreads out in model space, with many model elements lying far from any of the plausible rock-type clusters. The conventional regularized inversion, which takes prior information about these clusters into account, does a much better job of restricting model elements to the plausible part of model space, but is not successful in migrating most model elements to the correct clusters. The tunneling approach is effective in that it both restricts model elements to a priori plausible regions, but also allows for effective transfer of model elements between different clusters.

## Conclusions

Prior information can be a useful supplement to seismic data in full waveform inversion, improving both convergence rate and accuracy. Prior information about clustering of model parameters, however, is both often available and very challenging to treat with conventional full waveform inversion approaches. The tunneling approach we treated here augments the conventional, local optimization of FWI with an additional, non-local update that moves elements between clusters. In a synthetic test, this tunneling approach appears to have significant advantages over conventional treatment of clustering priors, and over neglect of available prior information.

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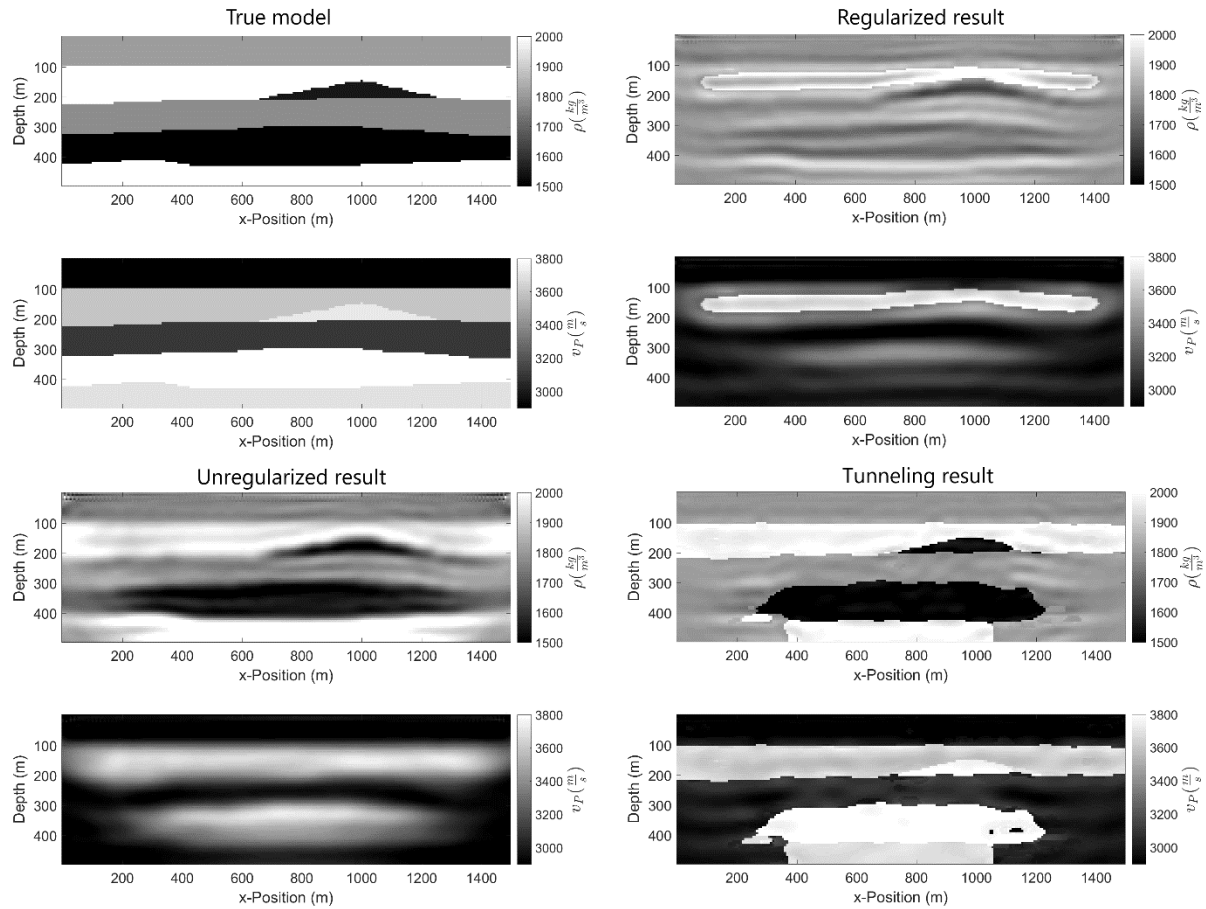


Figure 1. True model (top left) used in synthetic tests, as well as regularized (top right), unregularized (bottom left) and tunneling inversion (bottom right) results for  $v_P$  and density. The tunneling result improves on the accuracy and resolution of the conventional inversion strategies.

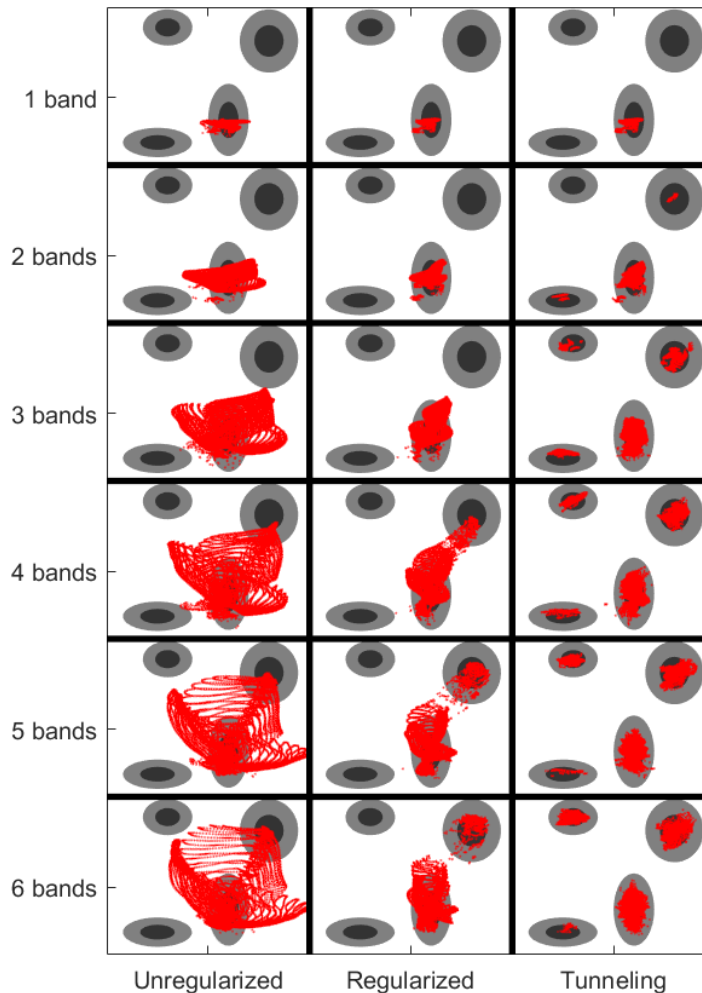


Figure 2. Density (x-axis) and P-wave velocity (y-axis) recovered for each model element at the first six iterations of an inversion. Conventional unregularized (left) inversion does not make use of prior knowledge of rock-property clusters (grey regions), and conventional regularized inversion (center) does not use this information effectively. The tunneling approach (right) allows for inversion updates which permit feasible model-space (grey regions) to be explored, without allowing the model to explore infeasible regions (white).