Incorporating multiple a priori information for full waveform inversion

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Summary

Full waveform inversion (FWI) is a high-resolution imaging technique for the seismic inversion problem. However, the inverse result is severely affected by the local minimum solution when the initial model is too different from the true model. To mitigate the local minimum issue, multiple a priori information of the problem can be introduced. When the a priori information are represented as convex sets, the inverse problem is a constrained optimization problem where the constraint set is the intersection of the convex sets. An optimization algorithm is proposed by this work to solve the above problem. The proposed algorithm is a combination of the scaled gradient projection method, a set expanding strategy, and the L-BFGS Hessian approximation. Numerical examples are provided and show that the inverse result can be improved when suitable a priori information is introduced.

Introduction

Compared to the conventional seismic inversion methods, the full waveform inversion (FWI) technique takes advantage of the "full" information of the seismic data, including travel time, amplitude, phase, time-frequency information, etc. So that it is expected to provide accurate and high-resolution information of the underground structures in the target area. The FWI technique was developed by Lailly and Tarantola in the early 1980s. In the FWI problem, the PDE that governs the seismic propagation, the recorded seismic data, the function of the seismic sources, and the locations of the receivers and the sources are known a priori. Given an initial model, the FWI problem is to minimize the difference between the seismic data simulated by the model and the recorded data. Then the inverse result will be updated iteratively by an optimization algorithm starting from the initial model. When the difference between the simulated data and the recorded data is decreasing, the model that generated the simulated data can be expected to become closer to the true model that generated the recorded data.

Incorporating the a priori information of the model to the FWI problem can lead to better inversion results. There are two equivalent ways to introduce the a priori information to the optimization problem: regularization and constraints. In the previous studies, the total variation (TV) constraint is used to reduce the cycle-skipping issues and build salt structures. Both box constraint and TV constraint are considered by several work. In the work (Peters, et al., 2019), the author developed an algorithm as a combination of the spectral projected gradients and Dykstra's algorithm, which can impose multiple constraints for the optimization problem. However, when the projection algorithm is evaluated inexactly, the update might be outside of the constraint sets and the constraint may not work in this case.

Method

In this work, we formulate the FWI problem as a nonlinear PDE constrained optimization problem through the reduced from. Consider the FWI problem:

$$\min_{y,u} J\,(y,u) = \frac{1}{2}||Qy - y_d||^2, \quad such\ that\ e(y,u) \ = \ 0.$$

Here the constraint PDE can be chosen as acoustic wave equation. The operator Q is the observation operator. Given N_c closed convex sets $U_i, i = 1, \cdots, N_c$ which representing the a priori information of the problem, the constraint set can be written as:

$$U_{ad} = \cap_{i=1}^{N_c} U_i,$$

where U_{ad} is nonempty. Since the constraint PDE is well-posed, the above FWI problem has a reduced form. Then the FWI problem with multiple a priori information can be written as:

$$\min_{u} f\,(u) = \min_{u} J\,(y(u), u), \quad such\ that\ u \in U_{ad}.$$

To solve the above constrained optimization problem, the scaled gradient projection method can be used. At the k-th iteration, the second order Taylor approximation of the objection function is:

$$f_k(u) = < \nabla f\left(u^k\right), u - u^k > + \frac{1}{2} < B_k\left(u - u^k\right), u - u^k >.$$

Here B_k is the Hessian approximation for the k-th iteration which is a symmetric positive definite matrix. Then, the scaled gradient projection method can be written as:

$$\widetilde{u^k} = u^k - B_k \nabla f\left(u^k\right),$$
$$\overline{u^k} = P_{B_k, U_{ad}}\left(\widetilde{u^k}\right),$$
$$u^{k+1} = u^k + \alpha_k\left(\overline{u^k} - u^k\right).$$

First, compute $\widetilde{u^k}$ which is the global minimum of the quadratic approximation $f_k(u)$. Then, project $\widetilde{u^k}$ towards to the set U_{ad} in the scaled Euclidean space generated by B_k. Update u^{k+1} with the line-search parameter α_k.

In our proposed optimization scheme, the B_k is generated by the L-BFGS Hessian approximation which provides an efficient Hessian and inverse Hessian approximation. To evaluate the projection $P_{B_k, U_{ad}}$, the projection onto convex sets algorithm by (Combettes, 2003) is introduced, that can work with both closed-form projection and subgradient projection. The Armijo line-search rule is used to determine the value of α_k. Furthermore, a special set expanding strategy is developed to maintain the global convergence of the algorithm. For a full description of the proposed optimization scheme, please refer to the work (Li, 2021).

Numerical example 1

A cross-well toy model is discussed in this example. The true velocity model and initial velocity model are given by Figure 1. The box constraint set is given by:

$$U_1 = \{u \in \mathrm{R}^{\mathrm{n}} \mid 1 \le u\_i \le 1.2\}.$$

The total variation constraint set is given by:

$$U_2 = \{u \in R^n \mid \left||u|\right|_{TV} \le 39.5\}.$$

The L1 constraint set is given by:

$$U_3 = \{u \in R^n \mid ||u - u_{ref}||_1 \le 128\}.$$

Here the reference model u_{ref} is the initial velocity model.

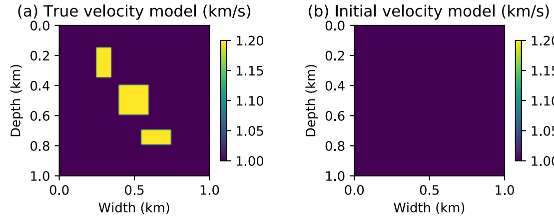

Figure 1: (a): True velocity model. (b): Initial velocity model.

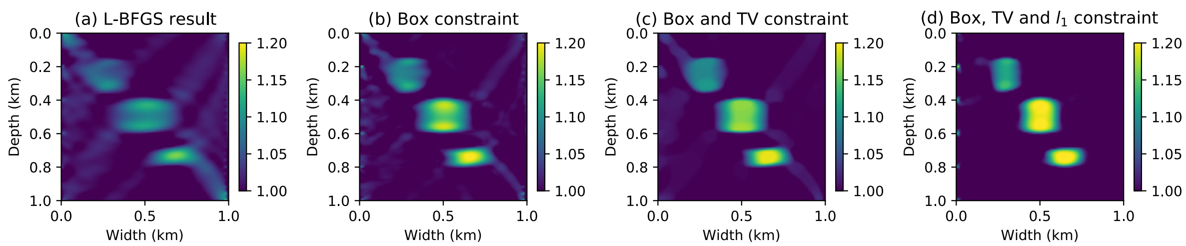

Figure 2: (a): Unconstrained result. (b): Inverse result with box constraint. (c): Inverse result with box and total variation constraint. (d): Inverse result with all three constraints.

Numerical results are shown in Figure 2. This example shows that the proposed method can handle multiple constraints at the same time. With more information provided for the optimization algorithm, a more accurate image can be achieved.

Numerical example 2

The Overthrust model example is provided in this section. There are 10 equally spaced sources and 126 equally spaced receivers on the top of the model. The true model and initial model are given in Figure 3.

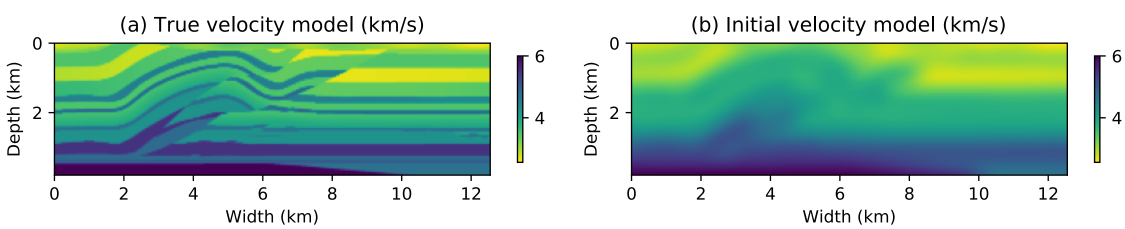

Figure 3: (a): True velocity model. (b): Initial velocity model.

In this example, we compare the inverse results with different size of the constraints. The box constraint is given by:

$$U_1 = \{u \in R^n \mid 2.5588 \leq u_i \leq 6\}.$$

The total variation constraints with different radius are given by:

$$U_2 = \{u \in R^n \mid ||u||_{TV} \leq 1200\}.$$
$$U_3 = \{u \in R^n \mid ||u||_{TV} \leq 1000\}.$$
$$U_4 = \{u \in R^n \mid ||u||_{TV} \leq 800\}.$$

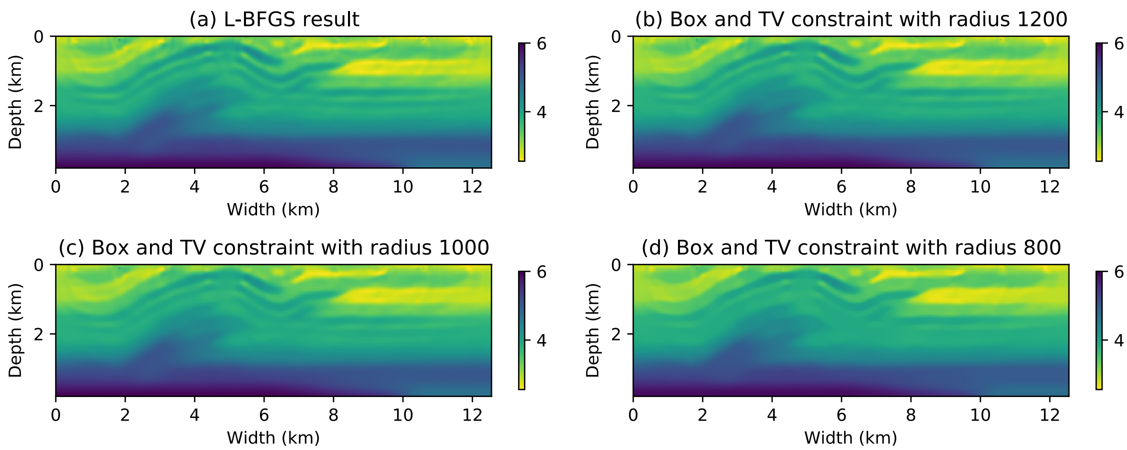

Figure 4: (a): Unconstrained result. (b), (c), (d): Inverse results with box and total variation constraints with different radius.

The inverse results are given in Figure 4. As the radius of the TV constraints is larger, the inverse result is closer to the unconstrained case. This example shows that the proposed optimization scheme can control the inverse result by changing the radius of the constraints.

Conclusions

A novel optimization scheme for the FWI problem that can incorporating multiple a priori information is proposed in this work. Numerical examples show that the proposed optimization scheme works with several kinds of constraints and is flexible to implement, and the inverse results of the FWI problem can be improved with proper a priori information.

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