Acoustic FWI using amplitude encoding strategy

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Summary

A super-shot or blended data strategy has been used in marine and land seismic surveys to reduce acquisition costs by reducing the time spent on the field. Full waveform inversion (FWI) has been used to estimate high-resolution subsurface velocity models. However, it suffers from expensive computational costs for matching the synthetic and the observed data. To reduce the costs of both data acquisition and processing, FWI using blended data has been recognized as very promising in future oil exploration. In this work, we use an amplitude encoding strategy with Hartley and cosine bases to accelerate the FWI process and compare their performance. The synthetic examples show that both two encoding functions can mitigate the crosstalk noise very well, providing good approximations of velocity models and convergence rate. To further improve the calculation efficiency, we also adopt the dynamic encoding strategy and reduce the number of super-shots every a few iterations. Since the encoding functions are not changed during iterations, we can directly simulate the super-shots without the blending stage. From the updated velocity model comparison, we can see that the inversion results by dynamic encoding are almost identical to those by static encoding.

Method

In conventional acoustic FWI, the objective function (data misfit function) is given by

$$E(\boldsymbol{m}) = \frac{1}{2}\boldsymbol{\Delta}\mathbf{p}^{\dagger}\boldsymbol{\Delta}\mathbf{p} = \frac{1}{2}||\mathbf{p}_{\text{cal}} - \mathbf{p}_{\text{obs}}||^2 \tag{1}$$

where $\Delta\mathbf{p}$, $\mathbf{p}_{\text{obs}}$ and $\mathbf{p}_{\text{cal}}$denote the data misfit, the recorded data and the forward modeled data, respectively. In amplitude encoding FWI, shot gathers are transformed into super shot gathers by the amplitude encoding matrix, which is defined as

$$\mathbf{B} = \begin{bmatrix} b^{1,1} & \cdots & b^{Nsig,1} \\ \vdots & \ddots & \vdots \\ b^{1,N_{sup}} & \cdots & b^{Nsig,N_{sup}} \end{bmatrix}_{N_{sup}\times N_{sig}} \tag{2}$$

where *Nsup* is the number of the super-shots and *Nsig* is the number of the individual shots (*Nsup* < *Nsig*). The *Nsig* synthetic data and observed data are blended into *Nsup* blended data $\mathbf{p}^{\text{sup}}$ by

$$\mathbf{p}^{\text{sup}} = \mathbf{B}\mathbf{p} \tag{3}$$

The ratio between *Nsig* and *Nsup* is the factor by which the computational cost is reduced. Since usually *Nsup* is much smaller than *Nsig*, the encoding FWI would achieve much better efficiency due to the reduction of data dimension. Then the encoding objective function is given by:

$$E(\mathbf{m}) = \frac{1}{2}\boldsymbol{\Delta}\mathbf{p}^{\dagger}\boldsymbol{\Delta}\mathbf{p} = \frac{1}{2}||\mathbf{p}_{\text{cal}} - \mathbf{p}_{\text{obs}}||^2 = \frac{1}{2}(p_{cal} - p_{obs})\mathbf{B}^{\text{T}}\mathbf{B}(p_{cal} - p_{obs}) \tag{4}$$

The matrix $\mathbf{B}^{\text{T}}\mathbf{B}$ is referred to as the crosstalk matrix, and when it's equal to the identity matrix, the encoding objective function is equal to the traditional objective function. FWI using blended data would produce the same results as in conventional FWI cases. In this work, we use Hartley and cosine bases as the encoding functions to design the amplitude encoding matrices. The Hartley encoding matrix is defined as (Tsitsas, 2010):

$$b_{m,n} = \cos\left(\frac{2\pi mn}{n_{\text{sig}}}\right) + \sin\left(\frac{2\pi mn}{n_{\text{sig}}}\right) \tag{5}$$

The discrete form of the cosine basis is (Hu et al., 2016):

$$b_{m,n} = \sqrt{\frac{2}{n_{\text{sig}}}} \cos\left(\frac{\pi}{n_{\text{sig}}} \frac{(2m\%n_{\text{sig}}+1)(2n+1)}{4}\right) \tag{6}$$

where m = 1,…, $Nsig$ is the shot-index, n = 1,…, $Nsup$ is the super-shot index, and $nsig$ is the periodization index, which we set to be half of $Nsig$.

Results

In this work, we use a Marmousi model to generate all synthetic shot gathers for 140 sources, which are evenly distributed near the surface of the true model with a spatial interval of 64 m. We deploy 576 receivers right beneath the sources with a spatial interval of 16 m. The Ricker wavelet sources are fired with a central frequency of 4 Hz. For conventional FWI, all the sources are fired individually and shot gathers are recorded separately. For amplitude encoding FWI, we apply different amplitude weights to the shot gathers to compose super-shots.

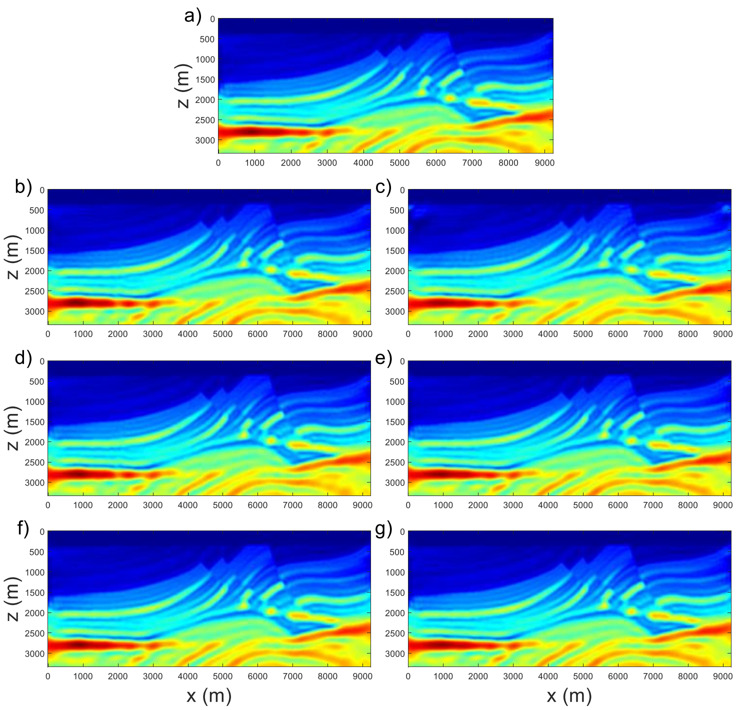

Figure 1. The updated velocity model after 100 iterations: a) by conventional FWI; b), d) and f) by amplitude-encoding FWI using Hartley basis with 7, 35 and 70 super-shots; c), e) and g) by amplitude-encoding FWI using cosine basis with 7, 35 and 70 super-shots.

In our experiments, we use both Hartley and cosine bases to design the encoding matrices. For comparison, we blend all the shot gathers into 7, 35 and 70 super-shots. With an increasing number of blended data, more off-diagonal elements of the crosstalk matrices are close to zero. In our experiments, we run FWI using a gradient-based method (Yang et al., 2015) for 100 iterations. For comparison, we also present the inversion result using conventional FWI. All inversion results are shown in Figure 1. When we first take a look at Hartley basis results (see

Figure 1b, 1d and 1f), we can notice there exists some crosstalk noise in the left middle, while with the increasing number of super-shots (from top to bottom), it can be better mitigated. And for the right column using cosine basis (see Figure 1c, 1e and 1g), the first image still contains some noise in the upper left, while the other two updated velocity models are almost noise-free. Generally, in our experiments, compared with the result by conventional FWI in Figure 1a, we can see using both amplitude encoding functions would also produce very good approximations of the velocity model, even with only 7 super-shots (see Figure 1b and 1c). While using more blended data would better mitigate the crosstalk noise with extra calculation effort. As shown in Figure 2, we compare the data misfit and we can notice that using amplitude encoding FWI shows very similar convergency as in the conventional case.

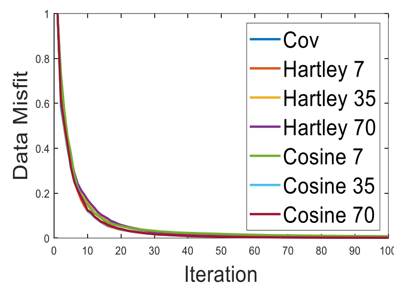

Figure 2: Comparison of data misfit function versus iteration using conventional FWI and amplitude encoding FWI.

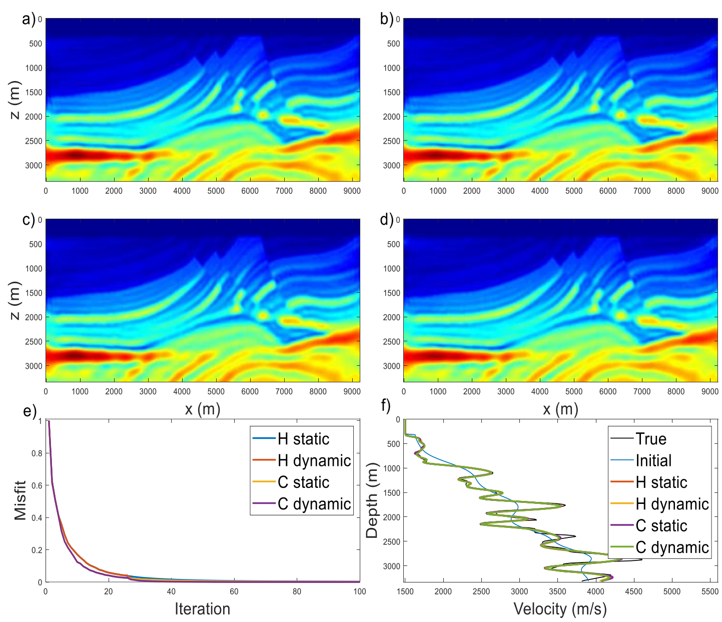

Figure 3: Comparison between dynamic and static encoding strategies: a) and b) are the inversion results by static amplitude encoding FWI using Hartley and cosine bases, respectively; c) and d) are the inversion results by dynamic amplitude encoding FWI using Hartley and cosine bases, respectively; e) is the comparison of misfit function versus iteration; f) is the comparison of vertical profile at the middle of the model.

To obtain ideal updated velocity models with very better mitigated crosstalk noise as shown in Figures 1f and 1g, the data dimension is not reduced enough. To further improve the calculation efficiency, we adopt the dynamic encoding concept (Krebs, 2009). In our case, instead of changing the encoding function, we reduce the number of super-shots every few iterations to further reduce the data dimension. In our test, at first, we still compose the shot gathers into 70 super-shots and run 25 iterations, then we compose the shot gathers into 35 super-shots and run another 25 iterations using the updated velocity model by the first step. Likewise, we then use 14 super-shots and 7 super-shots for 25 iterations each. So overall, we also update the velocity model 100 times. We present the inversion result comparison between the dynamic encoding and previous static encoding cases in Figure 3. When we respectively compare Figure 3a (same as Figure 1f) and 3c, 3b (same as Figure 1g) and 3d, we can see both encoding strategies provide almost identical inversion results and using both bases makes no obvious difference, but the data dimension has been further reduced. The data misfit and vertical profile comparisons are shown in Figures 3e and 3f, respectively. We can see that using dynamic encoding provides a very similar convergence rate as in the static encoding cases. Since the number of super-shots is changed every 25 iterations, the data misfit may not be smooth. From the comparison of vertical profiles in the middle of the model, we can also see all these 4 schemes give very good estimations of the true velocity model.

Conclusions

In this work, we present the amplitude encoding acoustic FWI using both Hartley and cosine bases as the encoding functions and compare their performance. In our experiments, for conventional FWI, it requires 140 forward model operations to generate the synthetic acoustic data. While for amplitude encoding FWI, we can directly simulate *Nsup* super-shots without the blending stage to improve the calculation efficiency for both forward modelling and the FWI inversion process. Both encoding functions can mitigate the crosstalk noise very well, providing good approximations of the velocity model and convergence rate. Using dynamic encoding would provide almost the same updated velocity models as in the static encoding cases, as well as further improve calculation efficiency considering the number of super-shots and iteration times.

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