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# 3D Seismic data reconstruction using an adaptive weighted rank-reduction method

*Farzaneh Bayati, Daniel Trad.*

*CREWES – University of Calgary*

## Abstract

A popular seismic data reconstruction method is rank-reduction of Hankel matrices generated from the data in the frequency domain using truncated SVD. However, the estimation of the Hankel matrix rank is complicated and affects the estimation of the singular vectors. We propose an adaptive algorithm that selects the rank in each frequency slice instead of using a predefined rank. The optimal rank is related to the criterion that defines the maximum difference between the energy of two adjacent singular values. The projection of the noise component to the signal component makes the results of the adaptive rank reduction less effective. We enhance the result of the interpolation by using a weighting operator that decreases the effect of noise and missing traces. We demonstrate the performance of the proposed method synthetic seismic data.

## Introduction

Seismic reconstruction methods can be divided into three main classes: signal processing-based methods, wave equation-based methods, and rank-reduction-based methods. In rank-reduction-based methods, the linear events in a clean seismic data set are low rank in the time domain. However, noise and missing traces increase the rank of data (Trickett, 2008). The rank reduction algorithm in the frequency domain is carried out in frequency slices by generating a Hankel matrix and applying a rank-reduction method on the generated matrix. The singular spectrum analysis (SSA) method proposed by (Oropeza & Sacchi, 2011) works by rank-reduction of the Hankel matrix with an iterative algorithm in the frequency domain. Gao et al., (2013) extended the SSA method to higher dimensional seismic data and called a multichannel singular spectrum (MSSA). One of the advantages of rank-reduction methods is simultaneous random noise attenuation and data interpolation. One of its limitations, on the other hand, is that it needs to meet the plane wave assumption. To satisfy the plane wave assumption the rank-reduction methods need to be applied on local windows. Most of the time it is not easy to find the proper window size because it is hard to decide whether the structure in the local window is linear or not. Moreover, it is hard to approximate the rank of each window. Choosing the wrong rank will lead to a failure because its overestimation will lead to a significant reconstruction error and its underestimation will distort the high frequencies. In this paper, we will apply a method that selects a rank automatically at each local window. The method was first proposed by (Wu & Bai, 2018) for 2D data. Then we develop the method by applying a weighting operator proposed by (Yangkang et al., 2019) on the selected singular values in each window to minimize the effect of noise component projection on the signal component projection. We employ the proposed rank selection criterion to synthetic 3D data and demonstrate its successful performance.

## Theory

Let us consider a seismic record in the time domain, $s(t,x,y)$, which after Fourier transform becomes $S(f,x,y)$. We generate a block of Hankel matrix for a constant frequency as follow:

$$\mathbf{M} = \mathcal{H}^{(2)}\mathrm{S}(f), \quad (1)$$

where $\mathrm{M}$ is the block Hankel matrix, $\mathcal{H}^{(2)}$ is the Hankelization operator and $\mathrm{S}(f)$ is the frequency slice of the input data. The next step of the SSA algorithm is rank reduction step with TSVD as follow:

$$\mathbf{M} = \mathbf{U\Sigma V}^{\boldsymbol{H}}, and \boldsymbol{\Sigma} = \mathrm{diag}(\hat{\sigma}_1, \dots, \hat{\sigma}_n), \quad (2)$$

where $\boldsymbol{\Sigma}$ is the matrix of singular values of $\mathbf{M}$. Matrices $\mathbf{U}$ and $\mathbf{V}$ contain as columns the singular vectors of $\mathbf{M}$. The values $\hat{\sigma}_i$ are the singular values in descending order. The number of non-zero singular values in the $\boldsymbol{\Sigma}$ determines the rank of the full-rank block Hankel matrix (Golub et al., 1971).

The next step is averaging the elements of the recovered block Hankel matrix to recover the signal in the Fourier domain. To reconstruct the amplitude of the missing traces completely, we can apply a POCS-like algorithm as below:

$$\mathbf{S}_{p+1} = \alpha_p \mathrm{S}_0 + (\mathbf{I} - \alpha_p \mathbf{T}) \odot \xi \mathbf{S}_p, \quad (3)$$

where $\odot$ denotes the Hadamard product of two matrices, $\mathrm{S}_0$ is the observed data, $\mathbf{S}_p$ is the reconstructed data after $p$ iterations, $\mathbf{I}$ represents an identity matrix, $\mathbf{T}$ denotes the sampling matrix which is one for observed data and zero for the missing traces, $\alpha_p$is a coefficient that changes linearly with the number of iterations, and $\xi$ is the SSA algorithm steps.

SSA uses the assumption of the plane waves which is unrealistic for real data. Complicated structures with large curvature are not suitable for rank-reduction methods like SSA. One solution for the curved events is to apply an NMO correction before the rank-reduction method. Another approach is to use local spatial windows to assume that events are linear. Even by applying the local spatial windows knowing the number of linear events is still challenging. In the SSA algorithm, as a rule of thumb, the rank of the block Hankel matrix equals the number of events in each local window that is equal to the number of distinctly large singular values (Aoki, 2013). The rule of thumb suggests that the optimal rank is equal to $r$ if:

$$\frac{\hat{\sigma}_{r+1}}{\hat{\sigma}_1} \leq \frac{1}{\sqrt{N}} \quad and \quad \frac{\hat{\sigma}_i}{\hat{\sigma}_1} > \frac{1}{\sqrt{N}}, \qquad i = 1, \dots, r \quad (4)$$

where $\mathrm{N}$ is the number of the linear events and $\hat{\sigma}_i$ is the $i$th singular value of the block Hankel matrix. In this paper, we are looking for the rank in a local window, whose number of events is unknown. When the number of events is undefined, we can find the desired rank from Equation (5) below. It satisfies the rule that the desired rank of complete noiseless data is related to the largest number of the non-zero singular values.

$$\Gamma(\boldsymbol{\Sigma}, r) = 3 \times \max_i \frac{\hat{\sigma}_i^2}{\hat{\sigma}_{i+1}^2}, \quad (5)$$

where $\Gamma(\boldsymbol{\Sigma}, r)$ indicates the operator that chooses the block of Hankel matrix in each frequency slice, and $r$ can be introduced as the optimal rank of the block Hankel matrix that minimizes the Frobenius-norm difference between the approximated and the exact signal components. Based on this criterion the desired rank of the block Hankel matrix in each frequency slice and each processing window depends on when the energy ratio between two singular values becomes the largest. The recovered block Hankel matrix in each frequency slice can be written as:

$$\mathbf{M}_r = \mathbf{U}\Gamma(\mathbf{\Sigma}, r)\mathbf{V}^H. \tag{6}$$

This approach leads to a satisfactory result for noiseless data; however, in the presence of random noise, the estimated signal component is still distorted by the noise component because of the projection of the noise component to the signal component. So, it is convenient to utilize a weighting operator that reconstructs the signal components effectively proposed by (Nadakuditi, 2013). The operator can be summarized as follow:

$$\widehat{\mathbf{W}} = diag(\widehat{w}_1, \widehat{w}_2, \ldots, \widehat{w}_r), \tag{7}$$

where $\widehat{w}_i = \left(-\frac{2}{\hat{\sigma}_i}\frac{D(\hat{\sigma}_i;\mathbf{\Sigma})}{D'(\hat{\sigma}_i;\mathbf{\Sigma})}\right)$, $D$ indicates the $D$-transform, and $D'$ denotes the derivative of $D$ with respect to $\hat{\sigma}_i$:

$$D(\hat{\sigma}_i; \mathbf{\Sigma}) = \left[\frac{1}{r}Tr(\hat{\sigma}_i(\hat{\sigma}_i^2\mathbf{I} - \mathbf{\Sigma}^2)^{-1})\right]^2, \tag{8}$$

$$D'(\hat{\sigma}_i; \mathbf{\Sigma}) = \left[\frac{1}{r^2}Tr((\hat{\sigma}_i^2\mathbf{I} - \mathbf{\Sigma}^2)^{-1} - 2\hat{\sigma}_i^2(\hat{\sigma}_i^2\mathbf{I} - \mathbf{\Sigma}^2)^{-2})\right]^2, \tag{9}$$

where $Tr$ (.) denotes the trace of input. The $D$-transform describes how the distribution of the singular values of the sum of the independent matrices is related to the distribution of the singular values of the individual matrices (Benaych-Georges, 2009). We can substitute the weighting algorithm obtained from Equation (7) in Equation (6) to enhance the results of the rank-reduction step as:

$$\widehat{\mathbf{M}}_r = \mathbf{U}\widehat{\mathbf{W}}\Gamma(\mathbf{\Sigma}, r)\mathbf{V}^H, \tag{10}$$

where $\widehat{\mathbf{M}}_r$ is the reduced rank block Hankel matrix and applying Equation (3) we can interpolate and denoise the 3-D data.

## Examples

We apply the methods to a 3-D synthetic shot gather with nine events. The data is a cube of 100 inline and 11 crosslines with SNR=4 and 60 percent gap. We choose the local window with 23 traces in the inline direction and 11 in the crossline direction for each method and set the number of iterations constant for all of them. The predefined rank for the traditional rank-reduction (TRR) method was set to 9 which equals the number of events in each window.

Figure 1 (a) shows a 3-D shot gather with SNR 4 and 60 percent missing traces arranged into a 2-D matrix. (b) indicates the clean and complete synthetic data. The results of applying the traditional rank-reduction (TRR), adaptive rank-reduction (ARR), weighted rank reduction method with predefined rank=9 (WRR), and adaptive weighted rank-reduction (AWRR) method are shown in Figure 1 (c), (e), (g), (i) respectively. Figure 1 (d), (f), (h), and (j) represent residuals errors for Figure 1 (c), (e), (g), and (i) respectively. All five methods recovered most of the signals. However, TRR remains serious residual, the result from ARR causes significant residual noise and AWRR is much cleaner than that of TRR, ARR and WRR.

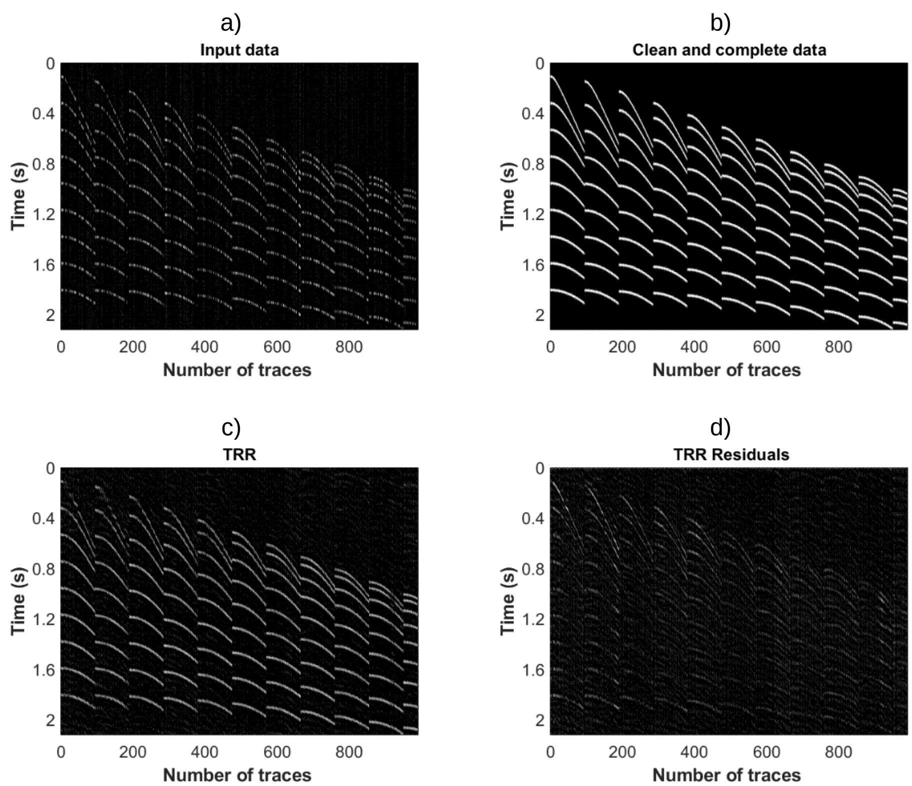
We selected a random trace from the reconstructed data of the different methods, from the clean data and from the noisy data in Figure 1 to examine the performance of each method in recovering the amplitude (Figure 2). We can see that the results of both ARR and AWRR are closer to the clean data than TRR and WRR. Moreover, the blue line that represents AWRR has less fluctuation than the other methods.

To evaluate the results of different rank-reduction approaches numerically, we use a quality factor defined as follows:

$$QF = 10log_{10}\frac{\|\boldsymbol{s}\|_2^2}{\|\boldsymbol{s}-\boldsymbol{d}\|_2^2}, \quad (11)$$

where $\boldsymbol{s}$ denotes the desired signal and $\boldsymbol{d}$ indicates the reconstructed data.

To test the performance of the different methods with different sampling ratios we examined the data in Figure 1 with SNR=4 with different sampling ratios from 10 percent to 70 percent. We can see in Figure 3 that all the methods are comparable when the missing traces are less than 40 percent. However, TRR fails for the sparse data. The result of the AWRR approach excels the other methods.



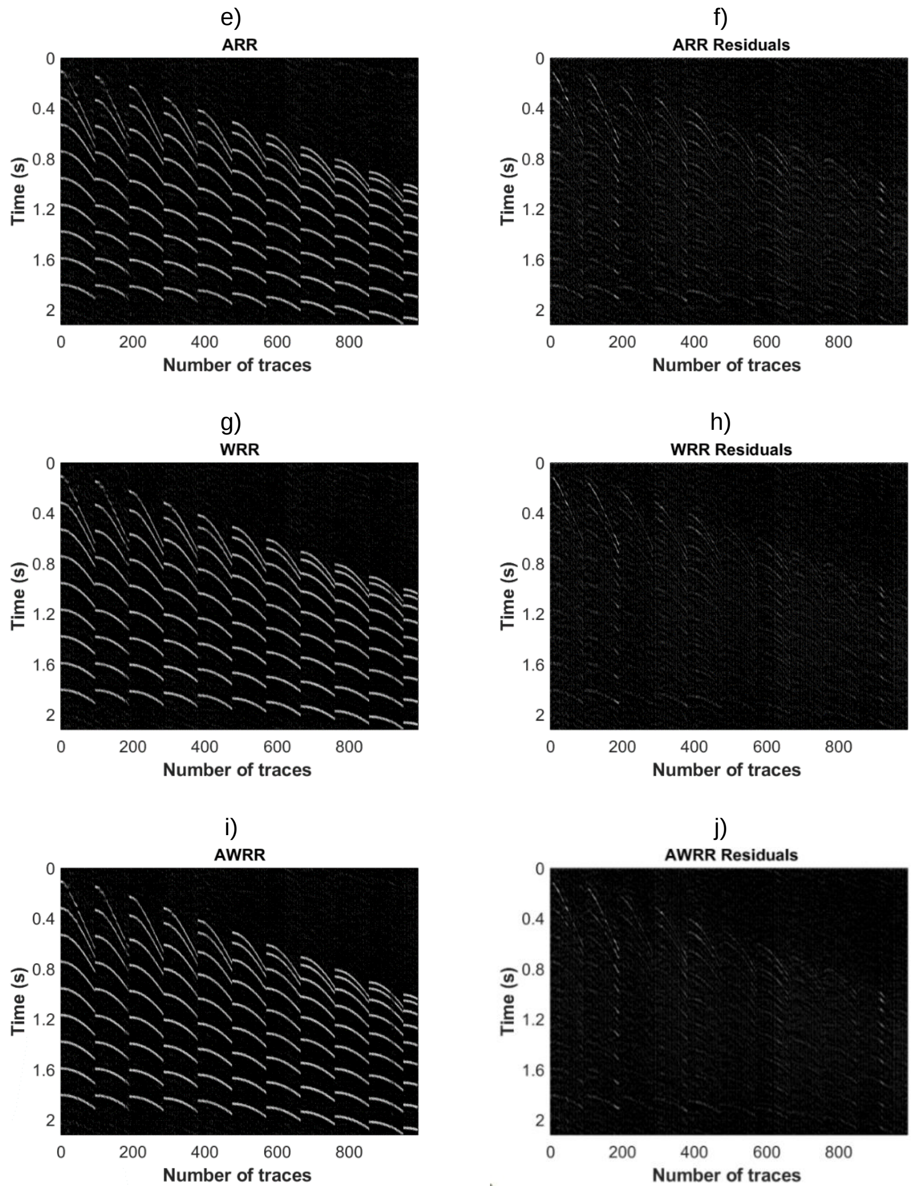



Figure 1: Applying different methods of rank reduction to a synthetic shot gather. (a) Input data with snr 4 and 60 percent gaps, (b) complete and clean data, (c), (e), (g), and (i) interpolated data using TRR, ARR, WRR, AWRR method, respectively. (d), (f), (h), and (j) Interpolation error of TRR, ARR, WRR, AWRR, respectively.

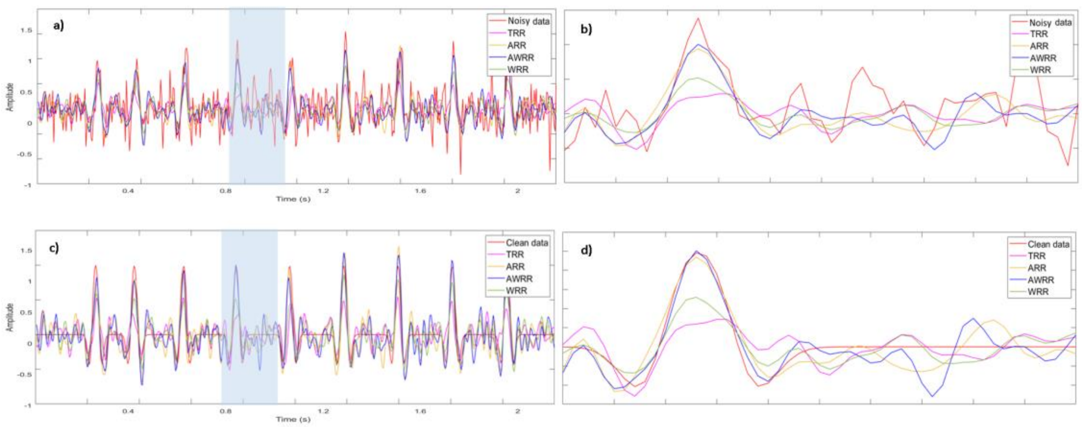



Figure 2: Amplitude comparison. (a) Comparison in the whole trace for the rank-reduction methods with the noisy data. (b) Zoomed area of the transparent blue window. (c) Comparison in the whole trace for the rank-reduction methods with the clean data. (d) Zoomed area of the transparent blue window.

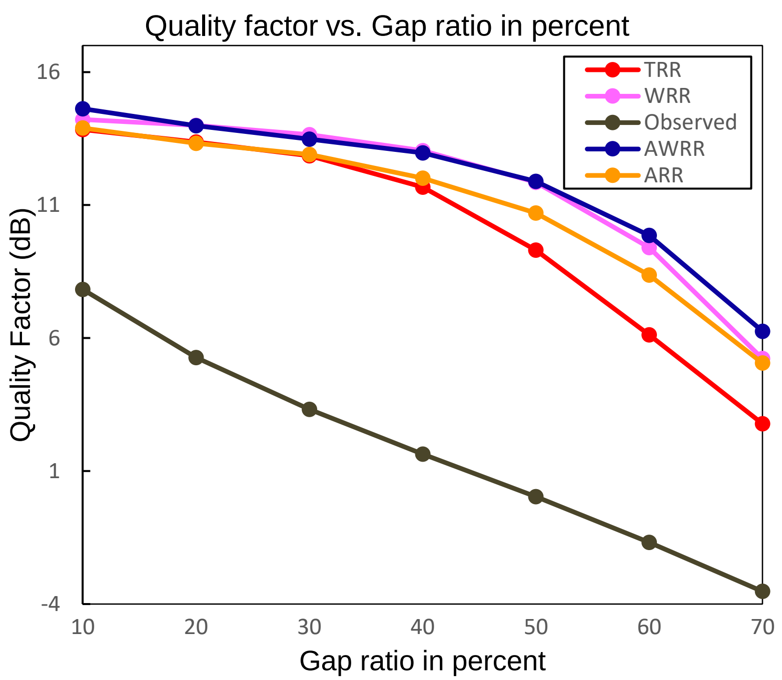



Figure 3: Diagram of the quality factor of the different methods with respect to the sampling ratio for the snr=4.

## Conclusions

We introduced a rank-reduction method for simultaneous denoising and interpolation of 3D data. The method first selects the optimum rank of the block Hankel matrix in each window of constant frequency and space and further improves the results by applying a weighting operator that reduces the effect of noise component projection from the signal component projection. The result of synthetic data shows that the proposed method is much closer to the true data than the other traditional methods and preserves the signal better than the traditional rank-reduction method.

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