geoconvention
Calgary • Canada • June 20-22 2022

Seismic reconstruction for compressive irregular-grid acquisition with I-FMSSA and EPOCS: A comparative study

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Summary

Spatially irregularly sampled seismic data is inevitable in field acquisition due to either natural obstacles or recently compressive-sensing design. Mapping seismic data from irregular-grid to regular-grid is a long-standing problem for seismic processing. We compare the extended POCS (EPOCS) method and interpolated-FMSSA (I-FMSSA) method for reconstructing compressive arbitrary irregular-grid acquisition data. A fast and computational efficient MSSA (FMSSA) algorithm is applied as the projection operator to accelerate the low-rank estimation. An interpolation operator is adopted to connect irregular-grid observations and desired regular-grid data for EPOCS and I-FMSSA methods. Synthetic and real data examples show the performance comparison of the EPOCS method and the I-FMSSA method for irregular reconstruction.

Introduction

The goal of seismic reconstruction is to regularize irregular-grid field data and increase the fold map for following seismic processing. In essence, there are two types of irregularity for field acquisition. One is natural irregularity. This kind of irregularity is caused by natural obstacles, such as lake, buildings, or high-way, which is inevitable and quite common during the field acquisition. Many reconstruction methods have been proposed to solve this problem. Liu and Sacchi (2004) introduce a minimum weighted norm interpolation (MWNI) method for multidimensional seismic reconstruction. Naghizadeh and Sacchi (2007) adopt prediction error filters for aliased regularly and irregularly decimated data reconstruction. Similarly, Zwartjes and Sacchi (2007) reconstruct aliased nonuniformly sampled data with sparse Fourier inversion. The other is human-made irregularity (compressive-sensing design). This irregularity is predefined for designing the acquisition geometry based on compressing sensing theory. Compressive sensing based seismic acquisition has been gaining importance in the oil industry due to economic cost and compressive ability (Herrmann, 2010; Li et al., 2012; Mosher et al., 2012). By utilizing the coherency of seismic signals in auxiliary domains, seismic records compressed to lower than Nyquist frequency can be recovered near-perfectly with sparse inversion. Jiang et al. (2017) extend the traditional POCS method (Abma and Kabir, 2006) to extended POCS (EPOCS) by incorporating an interpolation operator and extend its usage for under-sampled arbitrary irregular acquisition.

In the last decades, methods that exploit reduced-rank data approximations for irregularly decimated data reconstruction have been gaining popularity and interest. Rank-reduction constrained reconstruction methods can be divided into two categories: matrix-based reconstruction methods (Sacchi et al., 2009; Trickett et al., 2010; Oropeza and Sacchi, 2011) and tensor-based reconstruction methods (Kreimer et al., 2013; Kumar et al., 2015; Gao et al., 2015). Carozzi and Sacchi (2021) extend MSSA reconstruction method (Oropeza and Sacchi, 2011) to interpolated MSSA (I-MSSA) method for compressive irregular-grid data reconstruction by honoring the truce trace coordinate.

In this abstract, we compare EPOCS and interpolated-FMSSA (I-FMSSA) methods for compressive irregular-grid data reconstruction. A fast and memory-efficient MSSA (FMSSA) algorithm is adopted as a projection operator in conjunction with the projected gradient descent (PGD) method to accelerate the low-rank estimation. In the example section, we first compare the computation efficiency with the original MSSA algorithm and the FMSSA algorithm as the projection operator. For the synthetic and real data example, we compare the EPOCS method and the I-FMSSA method for compressive irregular-grid data reconstruction.

Theory

Extended POCS (EPOCS)

Before introducing the extended POCS (EPOCS) algorithm, we briefly describe the classical POCS method first introduced by Abma and Kabir (2006) for multi-dimensional seismic data interpolation and reconstruction. The POCS method for reconstruction problem can be solved iteratively by the following equation:

$$\mathbf{d}^{k+1} = \hat{\mathbf{u}} + (\mathbf{I} - \mathscr{T})\mathscr{S}^T \mathbf{T}_k \left(\mathscr{S}\mathbf{d}^k\right), \tag{1}$$

where $\hat{\mathbf{u}}$ is the observed data after binning $\hat{\mathbf{u}} = \texttt{Binning}(\mathbf{u})$, and $\mathscr{T}$ denotes the sampling operator, which can be easily proved that $\mathscr{T} = \mathscr{T}^*$ and $\mathscr{T}^*\mathscr{T} = \mathscr{T}$ (Liu and Sacchi, 2004; Naghizadeh and Sacchi, 2010; Cheng and Sacchi, 2015). Normally, the elements of a $N_x \times N_y$ matrix sampling operator are given by

$$\mathscr{T}_{ij} = \begin{cases} 1 & \text{if one trace is assigned to grid point } (i,j) \\ 0 & \text{if grid point } (i,j) \text{ is empty} \end{cases}. \tag{2}$$

Operators $\mathscr{S}$ and $\mathscr{S}^T$ denote forward and adjoint transform space (i.e., Fourier or curvelet transform). $\mathbf{T}_k$ represents the iterative hard thresholding operator with the threshold changing according to the iteration number. One popular hard thresholding model is decreasing the threshold value exponentially according to the iteration number (Gao et al., 2013):

$$\mathtt{T}_k(x) = \begin{cases} x, & |x| \geq t_k \\ 0, & |x| < t_k \end{cases} \tag{3}$$

where, $t_k = t_{\max} e^{c(k-1)/(N-1)}$, and $c = \ln\left(t_{\min}/t_{\max}\right),\ k = 1, \cdots, N.$

The classical POCS method requires binned input on the regular grids. For the irregular-grid data, especially for CS-designed irregularity, the POCS method will cause amplitude and phase distortion, as the binning process directly assigns the arbitrary trace into its nearest regular-grid position. Therefore, the Extended POCS (EPOCS) method was an appropriate alternative for solving the arbitrary irregular-grid data (Jiang et al., 2017), which can effectively avoid the binning error.

For the irregular-grid data, the EPOCS method can be written by modifying equation 1 as the following expression:

$$\mathbf{d}^{k+1} = \mathscr{W}^*\mathbf{u} + (\mathbf{I} - \mathscr{W}^*\mathscr{W})\,\mathscr{S}^T \mathbf{T}_k \left(\mathscr{S}\mathbf{d}^k\right) \tag{4}$$

where $\mathscr{W}$ denotes the interpolation operator that maps data from regular grid to irregular grid, and $\mathscr{W}^*$ is its adjoint counterpart that maps from irregular grid to regular grid. $\mathbf{u}$ denotes the observed irregular-grid data that obey the true coordinates, and $\mathbf{d}$ represents the desired regular-grid data.

Interpolated-FMSSA (I-FMSSA)

The rank reduction method is also a useful tool for reconstructing irregularly decimated samples, which has been gaining popularity and interest in the last decade. However, the conventional Multichannel Singular Spectrum Analysis (MSSA) algorithm can be expensive to construct Hankel structured matrices and Singular Value Decomposition (SVD), especially for multi-dimensional data. Therefore, we propose to use fast and computational efficiency MSSA (FMSSA) as a substitution of conventional MSSA (Cheng et al., 2019), in order to speed up the rank-reduction procedure.

FMSSA

The MSSA algorithm is an accurate method for low-rank estimation. However, the main drawbacks of the MSSA algorithm are: 1) Building Hankel matrices induce computational complexities and increases memory load. 2) MSSA applies SVD on large matrices, which could be expensive for multi-dimensional problems.

In order to avoid the disadvantages mentioned above, we adopt a similar strategy from Cheng et al. (2019) to achieve a fast and computational-efficient MSSA (FMSSA) algorithm with the following modifications:

1. *Hankel matrix-vector products are computed via FFT to avoid Hankel structure matrices.*
2. *Instead of applying SVD to the Hankel matrix, randomized QR decomposition (rQRd) is adopted as an alternative for fast approximation.*
3. *Convolution is adopted to accelerate the anti-diagonal averaging.*

I-FMSSA

For irregular-grid data, low-rank constraint is another alternative for reconstruction. Normally, irregular-grid data reconstruction can be solved with the following cost function:

$$J = \|\mathbf{u} - \mathscr{W}\mathbf{d}\|_2^2 \quad s.t. \;\; rank(\mathbf{d}) \leq k \tag{5}$$

where $\mathbf{u}$ denotes the observed irregular-grid data, and $\mathbf{d}$ represents the desired regular-grid data.

The Projected Gradient Descent (PGD) method (Cheng and Sacchi, 2016; Lin et al., 2021; Carozzi and Sacchi, 2021) can be adopted for solving equation 5:

$$\mathbf{d}^{k+1} = \mathscr{P}\left[\mathbf{d}^k - \lambda \mathscr{W}^*(\mathscr{W}\mathbf{d}^k - \mathbf{u})\right] \tag{6}$$

Where $\mathscr{P}$ is the projection operator, which can be chosen as FMSSA algorithm, and $\mathscr{W}$ and $\mathscr{W}^*$ are interpolation operator pairs and have the same meaning as the equation 4.

Examples

We first compare the reconstruction quality of the I-FMSSA method (when projection operator $\mathscr{P} =$ FMSSA) with the I-MSSA method (when projection operator $\mathscr{P} =$ MSSA) for reconstruction. We synthesize an example containing three dipping linear events to mimic a small 3D patch of common receiver gather. The regular grid consists of 30×30 source points with interval $\Delta x = \Delta y = 20$m in the $x-$ and $y-$ directions, and a Ricker wavelet of central frequency 20 Hz was adopted. I added a perturbation to the regular grid to achieve the irregular distribution. The geometry of source coordinate of irregular-grid and desired regular-grid distribution is displayed in Figure 1a and 1b, respectively.

Figure 1e shows the reconstruction result with the I-MSSA method. We observe a high-quality reconstruction result (SNR = 20.6 dB) for this synthetic data. Similarly, the reconstruction result with the I-FMSSA method can be found in Figure 1g, in which the reconstruction result is incredibly same as Figure 1e with SNR = 20.8 dB. However, the computing time for this synthetic data shows a huge difference. Adopting the I-FMSSA method (time = 7.71s) needs much less computational time than the I-MSSA method (time = 31.92 s). We conclude that the I-FMSSA method is an efficient and accurate alternative for the I-MSSA method when applying for the reconstruction problem. Figure 1i depicts the reconstruction result with the EPOCS method. Comparing Figure 1j with 1f and 1h,

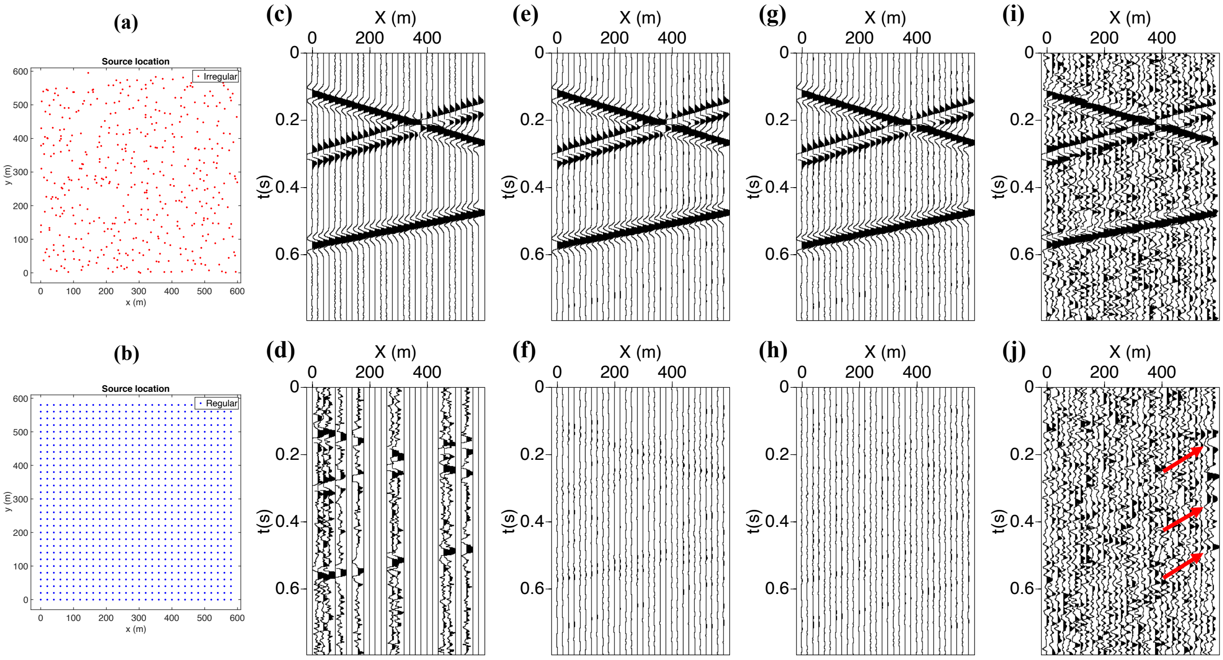

Figure 1: Irregular-grid reconstruction result comparison. (a) Coordinates of the irregular gird with 50% decimation. (b) Desired regular grid of output. (c) Clean data. (d) Noisy irregular-grid observed data. (e) Reconstruction with I-MSSA method with $SNR = 20.6\ dB$. (f) Difference between (c) and (e). (g) Reconstruction with I-FMSSA method with $SNR = 20.8\ dB$. (h) Difference between (c) and (g). (i) Reconstruction with EPOCS method with $SNR = 3.5\ dB$. (j) Difference between (c) and (i).

we observe that the EPOCS method fails to remove the random noise with SNR = 3.44 dB, also, produces non-negligible signal leakage at the boundary of the seismic profile.

We also compare the methods with a small patch of field data from the West Canadian Basin. The selected survey area contains 113 source points, and the mean intervals between sources and source lines are 100 m and 300 m, respectively. The desired regular-grid intervals between sources and source lines are 100 m and 150 m, respectively, including $10 \times 29 = 290$ source points. The coordinate distribution can be found in Figure 2a and 2b, respectively. The results of the EPOCS method for an inline slice and cross-line slice can be found in Figure 2e and Figure 2f, respectively. The similar results of the I-FMSSA method can be found in Figure 2g and 2h. We observe that both methods are effective for irregular-grid data reconstruction, and the empty traces are fully reconstructed. Beyond that, except for reconstruction, the I-FMSSA method is also valid for random noise attenuation.

Conclusions

This abstract illustrates a comparison for compressive arbitrary irregular-grid acquisition data reconstruction with EPOCS and I-FMSSA methods. The I-FMSSA method shows significant improvement in computational efficiency when the FMSSA algorithm is adopted as the projection operator compared with the I-MSSA method. Both EPOCS and I-FMSSA methods are effective for empty traces reconstruction. The EPOCS method shows signal leakage of the boundary traces and fails to attenuate the random noise for the synthetic example. On the contrary, the I-FMSSA method can achieve slightly cleaner and stable reconstruction results for noisy data. However, it does not mean that the EPOCS method can be replaced with the I-FMSSA method when dealing with irregular-grid reconstruction problems, as we believe that it is feasible to attenuate the random noise by following a denoising algorithm after EPOCS reconstruction. In addition, compared with random noise attenuation, geophysicists are more worried about the signal attenuation or leakage.

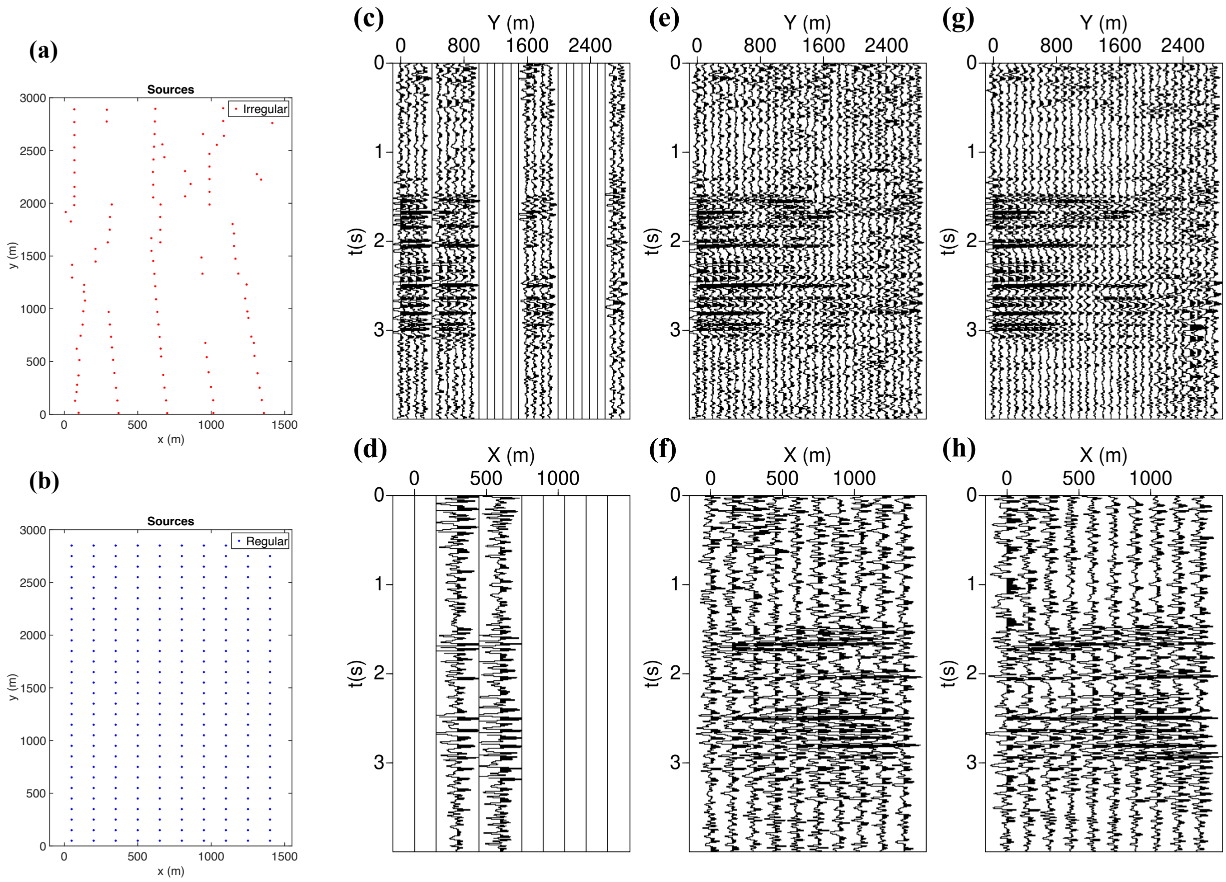

Figure 2: Real data irregular-grid reconstruction result comparison. (a) Coordinates of observed irregular gird, including 113 sources points. (b) Desired regular grid of output, containing $10 \times 29 = 290$ source points. (c) Observed inline irregular-grid data. (d) Observed crossline irregular-grid data. (e) Inline reconstruction with EPOCS method. (f) Crossline reconstruction with EPOCS method. (g) Inline reconstruction with I-FMSSA method (h) Crossline reconstruction with I-FMSSA method.

Acknowledgments

The author would like to thank the sponsors of the Signal Analysis and Imaging Group at the University of Alberta.

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