# Hypocenter locations: Semblance-weighted stacking versus traveltime inversion

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## Summary

Hypocenter locations are one of the key outcomes of microseismic data processing. Typically, these are estimated using one of the traveltime inversion, waveform stacking or migration-based approaches. One of the key requirements of the traveltime approach is that the P and S arrival times are picked/available for a reasonable number of receiver levels which is not always possible due to the strong noise or weak signal in the input waveforms. The waveform complexity also prevents reliable arrival picking. Furthermore, the performance of most if not all automatic pickers deteriorates for noisy waveforms and hence more manual effort is required to produce considerably accurate picks. The waveform stacking approach, on the other hand, does not require a comprehensive arrival picking effort and might be advantageous in the case of noisy waveforms. Here, we compare the semblance-weighted stacking and traveltime inversion approaches for hypocenter location determination using synthetic data examples. Our comparison is focused on the arrival picking effort (speed) and accuracy of the resulting hypocenter locations.

## Hypocenter location determination

### *Traveltime inversion*

Traveltime inversion requires picking of P and S-wave arrivals on the observed event waveforms. In this method, these picked arrival times are compared with the modeled traveltimes computed for each receiver level from different source positions using a calibrated velocity model. The position that minimizes the traveltime residuals is considered as the estimated hypocenter location. Geiger's method (1912) is a popular linearized inversion method that iteratively updates the hypocenter location from an initial guess by solving

$$\mathbf{G}\Delta\mathbf{m} = \Delta\mathbf{r}\,, \tag{1}$$

where $\mathbf{G}$ contains traveltime derivatives with respect to the model parameters $\mathbf{m} = (\mathrm{x},\ \mathrm{y},\ \mathrm{z},\ \tau_0)^{\mathrm{T}}$. $\Delta\mathbf{m}$ represents the perturbation vector for the model parameters and $\Delta\mathbf{r}$ is a vector containing the traveltime residuals (Akram, 2020). The damped least square solution of the eq (1) can be written as follows

$$\Delta\mathbf{m} = \left(\mathbf{G}^{\mathrm{T}}\mathbf{G} + \lambda\mathbf{I}\right)^{-1}\mathbf{G}^{\mathrm{T}}\Delta\mathbf{r}\,, \tag{2}$$

where $\lambda$ is the damping parameter or Tikhonov regularization coefficient. The model parameters are updated iteratively ($\mathbf{m}_{\mathrm{k+1}} = \mathbf{m}_{\mathrm{k}} + \Delta\mathbf{m}$) until a stopping criterion is met. Another way of determining hypocenter location is to use an exhaustive-grid search approach (Rodi, 2006). In this method, a look up table is generated using the modeled traveltimes for a pre-defined gridded zone, followed by the evaluation of the cost function (typically an $L_2$-norm) for the residuals between the observed and modeled traveltimes. The hypocenter location is determined from the spatial location of the grid point with minimum value of the cost function (Akram, 2020).

### *Semblance-weighted stacking*

The waveform stacking method requires very little to no effort on P and S arrival picking (Eaton et al., 2011; Zhang and Zhang, 2013). For a given time window *t*, the semblance can be calculated as follows (Kennett, 2000; Eaton et al., 2011)

$$\sigma(t) = \frac{\sum_k w_k (\sum_M u_{Mk})^2}{N \sum_k w_k \sum_M u_{Mk}^2}, \quad (3)$$

where $N$ represents the number of receivers and $w$ is a weighting function. $\Sigma_M$ is the summation over all receiver levels whereas $\Sigma_k$ is the summation over the time gate *t*. For strong coherent events, the semblance value is close to 1 (Kennett, 2000). Using the semblance, a weighted stack can be calculated as (Eaton et al., 2011)

$$S = \sum_m \sigma_m^q \sum_M \sum_k u_{Mkm}, \quad (4)$$

where $q$ is a user-defined exponent and $\Sigma_m$ is the summation over the three waveform components. The semblance values are computed for both P and S-wave data and their product is considered as the cost function value for a given location. This process can be repeated for all grid nodes in a pre-defined zone and the hypocenter location is determined from the spatial location of the grid point that has the maximum semblance stack value.

## Results and Discussion

Figure 1 shows an example of synthetic 3C waveforms recorded on a 12-level receiver array. A staggered-grid finite difference method is used to model the waveforms for a source located at 479 m offset from the receiver array. The modeled traveltimes as well as the time gates used in the semblance calculations highlight the P and S-wave arrivals.

Figure 2 shows only the z-component waveforms after the addition of different perturbations of white Gaussian noise. We pick the P and S arrival times using a fuzzy c-means and AIC based workflow (Akram et al., 2022). As the signal-to-noise ratio of the 3C waveforms decreases, the accuracy of the picked arrival times also decreases and at some point, the picking algorithm is not able to produce any practical results. Some of these picking errors can be minimized by a manual quality control process. However, for very low signal-to-noise ratio waveforms, accurate identification of the onset time becomes almost impossible, even with the manual quality control. If used further in the analysis, the resulting hypocenter locations will exhibit the effect of these arrival picking errors. On the other hand, the effort required by the semblance method is relatively small as even a rough guess of the arrival pick on any of the top receiver levels in the noisy waveform example is sufficient to setup the moveout corridors and determine the hypocenter location. By computing the semblance in a moving window fashion, we can even eliminate the need of picking arrival on one of the receiver levels. This, however, adds to the computational cost.

We manually correct the arrival picks on one of the noisy perturbations (zoomed view in Figure 2) of the synthetic waveforms and use the arrival picks and back-azimuth angles to estimate the hypocenter location (Figure 3). Because of the noise in the waveforms, the estimates of back azimuths also contain errors and consequently affect the accuracy of the hypocenter location. For the semblance-weighted stack, we rotate the waveforms into ray-centered coordinates using the azimuth, inclination angles from the modeled ray paths before applying the stacking. This minimizes the role of waveform based back-azimuth estimation but adds to the computational cost since for each grid node the waveforms are rotated for every event (Figure 3).

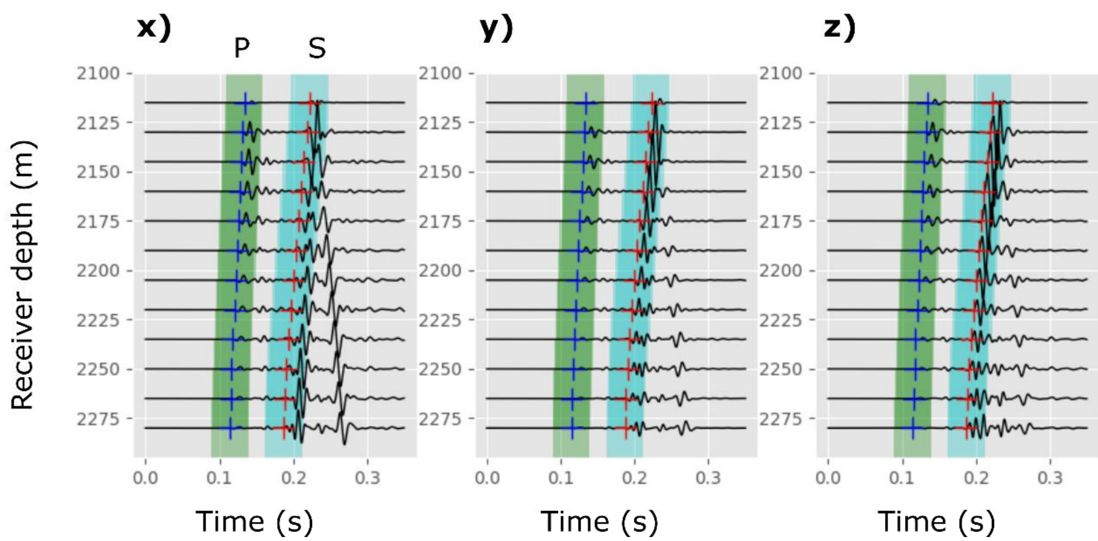


*Figure 1: 3C waveform examples of a synthetic microseismic event. The time gates for semblance calculation as well as the modeled traveltimes for both P and S arrivals are shown.*

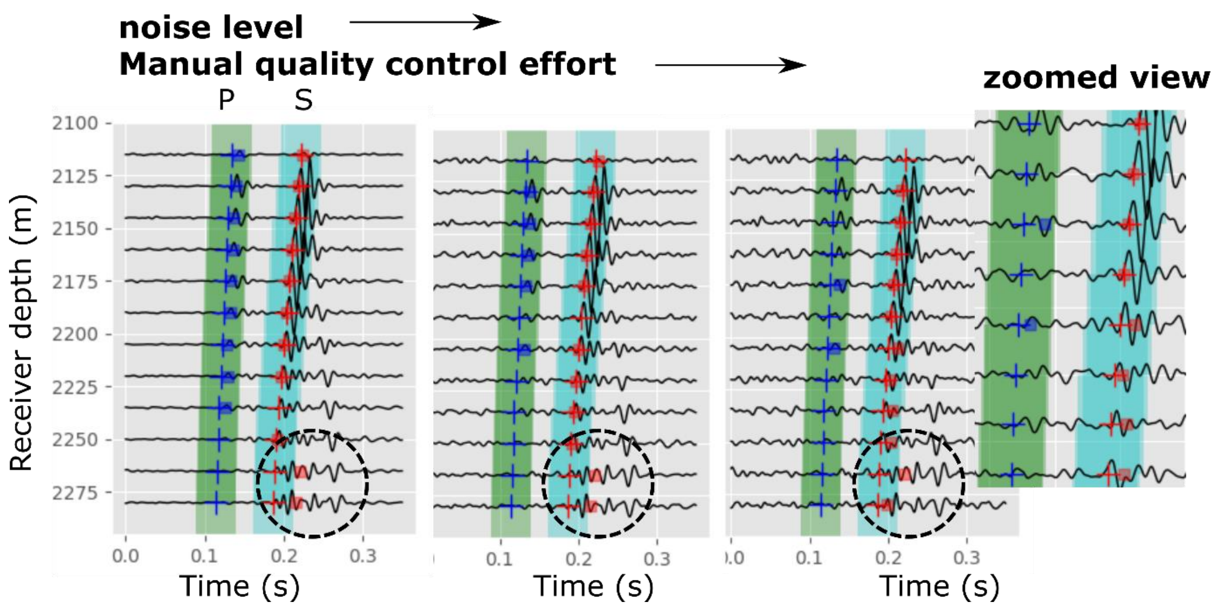


*Figure 2: z-component of the waveforms from Figure 1 with the addition of Gaussian noise. Autopicking results from the fuzzy cmeans-AIC method are shown (squares) along with the modeled times (+) for comparison. The circles highlight the S-wave mispicked results.*

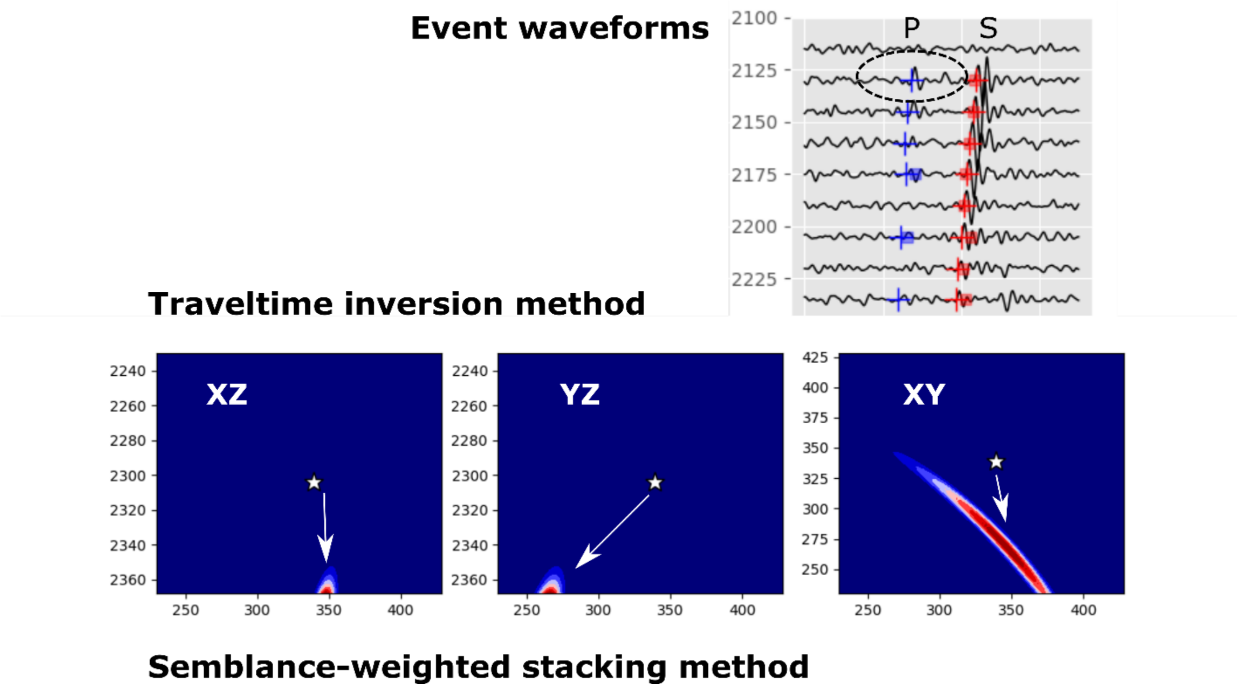


*Figure 3: Example of the noisy waveforms for a microseismic event. The ellipse shows the arrival used as reference for the time gate. Manually corrected picks (+) are shown for comparison with the autopicking results (squares). The hypocenter locations determined using the traveltime inversion and semblance-weighted stacking approach is also shown. The true location is represented by the star sign.*

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### References

Akram, J. 2020. Understanding downhole microseismic data analysis: With applications in hydraulic fracture monitoring, Springer.
Akram, J., Peter, D., and Eaton, D., 2022, Application of a fuzzy c-means and AIC based workflow for P/S arrival-time picking on microseismic data: Geoconvention, Calgary, Canada (submitted for review).
Eaton, D., Akram, J., St-Onge, A., and Forouhideh, F., 2011, Determining microseismic event locations by semblance-weighted stacking: Recovery – 2011 CSPG CSEG CWLS Convention, Calgary, Canada.
Geiger, L. 1912. Probability method for the determination of earthquake epicenters from the arrival time only. Bulletin of the St. Louis University 8: 60–71.
Kennett, B. L. N., 2000, Stacking three-component seismograms: Geophysical Journal International, 141, 263-269.
Rodi, W. 2006. Grid-search event location with non-Gaussian error models. Physics of the Earth and Planetary Interiors 158: 55–66.
Zhang, W., and Zhang, J., 2013, Microseismic migration by semblance-weighted stacking and interferometry: SEG Annual Meeting, Houston, USA, 2045-2049.