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First arrival travel time modeling using random grids

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Summary

We present a ray tracing algorithm based on the shortest path method that models the velocity field using random grids iteratively. These random grids adapt from one iteration to the next, selectively densifying the grid in areas with high ray density, preventing systematic errors generated using regular grids. By focusing on areas close to the actual trajectory of the ray, it is possible to minimize the error faster than in conventional implementations of the shortest path algorithm without significantly increasing the computational cost. We use the algorithm to model first arrivals with straight rays in a constant velocity field, showing the accuracy and efficiency of the method. The algorithm is four times faster than the Multiple Graph Realization method, which also relies on stochastic grids. Finally, an example including complex topography and its corresponding optimized network is shown.

Introduction

Ray tracing can be used in geophysics to model the interaction of seismic waves with the subsurface, and is necessary to solve geophysical inverse problems such as earthquake hypocentre location, tomographic inversion, migration, etc. In the case of tomographic inversion, the forward modeling can be the most computationally expensive part of the process (Moser, 1991), which is why finding efficient ways to trace rays is relevant. Some of the most computationally efficient methods for ray tracing are the finite difference solution to the eikonal equation (Vidale, 1988) and those based on graph theory (Nakanishi and Yamaguchi, 1986). We decided to work with the shortest path method due to its three main advantages: 1) The shortest path method always arrives at the global solution for any given graph map, 2) it is stable, and unaffected by high impedance contrasts that may cause other methods to fail and 3) there is no notion of dimensionality, which allows handling 2D or 3D geometries using the same algorithm (Moser, 1991).

Graph theory and ray tracing

In Graph Theory, a graph is a set of objects (nodes) connected by weighted links (edges) (Tutte and Tutte, 2001). To relate this abstract mathematical definition with geophysics, we may think of a set of nodes that sample the velocity field representing points in the subsurface, where the weights of this network are the travel times between nodes (the euclidean distance divided by the internodal velocity). Since the seismic ray is stationary and its travel time should be minimum (Fermat's principle), we can reduce the ray tracing problem to a shortest path problem in graph theory.

The shortest path problem is used to search for the minimum weight route between two nodes. This problem can be solved using Dijkstra's Algorithm (Dijkstra et al., 1959), which provides the shortest path between a "source" node, to all the other nodes of the network (similar to the way wave propagation works).

Another critical aspect of the algorithm is to define how the travel time queue will be handled. Different data structures to handle queues are available, but according to Moser (1991), the use of the minimum binary heap is the most efficient. Several tests were carried out, and we decided to incorporate the binary min-heap into our algorithm. A binary heap is a data structure that orders an array in

a tree-like structure following one simple rule; every value has to be equal to or smaller than the two following values (in the case of a minimum heap). In this way, we always ensure that the minimum weight is efficiently retrieved from the travel-time array by taking its first element.

Grid's geometry

A complete analysis of the effect of the geometry of the grid can be found in Bogiatzis et al. (2021), which analyzed the impact of using regular grids, circular grids, circular rotated grids, and irregular grids, concluding that irregular grids circumvent the azimuthal sampling problem of traditional grids, lowering the error for networks with the same amount of nodes. It also introduces the concept of Multiple Graphs Realizations (MGR), which calculates travel times on random grids iteratively. Then the resulting field is interpolated to some primary nodes. This method requires several iterations; in each one, it minimizes and replaces the travel time to a primary node. Finally, it optimizes the entire travel time field until meeting convergence criteria.

Iterative ray tracing on random grids

We will take advantage of the MGR technique and its random spatial sampling but increasing the number of nodes around the initial ray guess iteratively, as proposed by Sekiguchi (2021). In this case we can increase the spatial sampling around the rays of interest, but the nodes will be sparse for the rest of the network, keeping computational times small but with high accuracy. We propose the following workflow: initially, we generate a random mesh with a fixed number of nodes, and trace the rays of interest. In the second iteration, we initialize the mesh with another random grid and include the previous trajectory of the rays, perturbing and minimizing its traveltime. With every iteration, the mesh becomes denser around the ray of interest. This is particularly useful when you need to model a specific subset of the first arrival wavefront (like the first arrivals in the seismic refraction experiment) and when different rays share part of their trajectory. Another advantage is that using this approach, we can build a "memory" of which nodes are necessary to trace certain rays, which helps increase the convergence rate . In figure 1, we show a sketch of the method.

Results

To analyze the presence of cumulative and systematic error with distance in the ray tracing, we followed the example shown by Bogiatzis et al. (2021). We defined a 400km x 400km constant velocity model (7km/s), and we will optimize the travel times for a regular grid of 250000 nodes. The ray tracing is done by iterating a random network of 5464 nodes generated at each iteration. The main difference in our approach is that the network that minimizes the travel time to the reference nodes (desired rays) is stored from one iteration to the next, making the network more precise at each iteration and increasing the convergence speed.

The error was minimized to less than 10ms after 25.5 seconds of computing time, which is four times faster than other stochastic methods such as MGR (Bogiatzis et al., 2021). To illustrate the effect of node self-densification, we simulated an acquisition on a complex topography profile with three layers with variable morphology. Figure 3 shows the reference velocity model and the ray tracing for one source. It also shows the optimized network after five shots. We can see that the stored nodes are mapping the high-impedance contacts.

Conclusions

We have presented a different approach to the shortest path ray tracing method for cases when we desire to model a finite number of rays and not the entire travel time field. The algorithm outperforms classical approaches to the shortest path method and newer stochastic methods such as MGR. The error maps showed no regular error patterns, which shows that the algorithm's precision is not

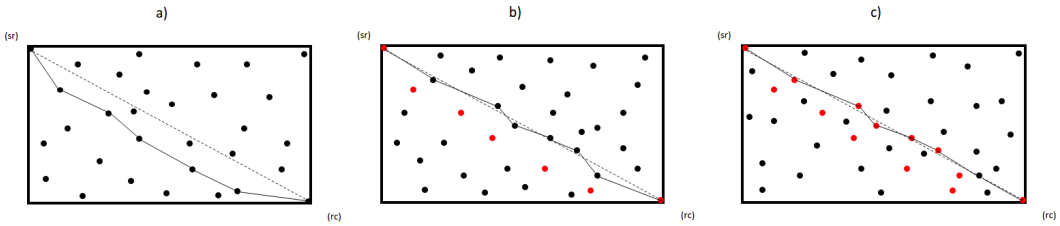

Figure 1: The diagram shows iterations of the SARG-SP method. We are trying to find the shortest path between point sr and point rc. a) First random network, in black, is the initial guess for the ray's trajectory. b) the second iteration. Notice how the initial ray nodes are stored and included in the second network. c) Final Iterations. All nodes from previous iterations are stored, making the network denser around the actual trajectory of the ray. Notice how the modelled ray approximates the real shortest path with each iteration (dashed line).

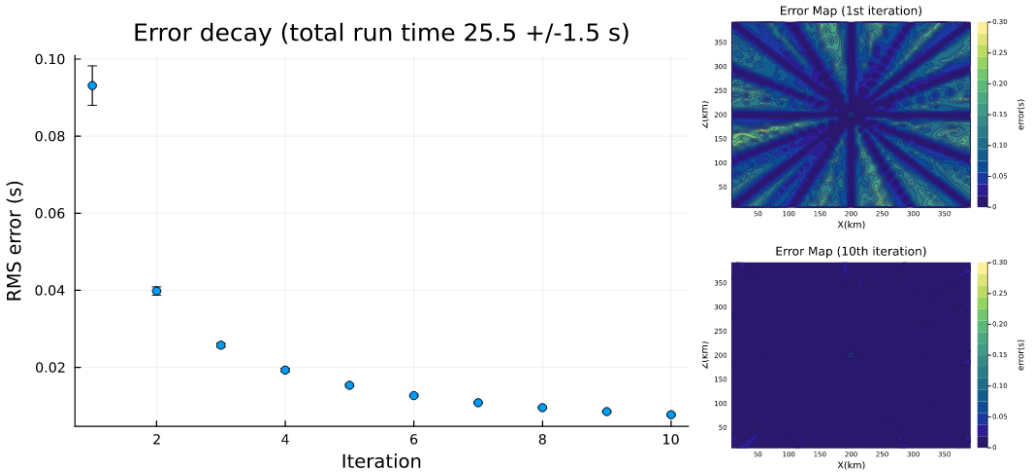

Figure 2: Error decay graph and maps. The error decay shows that it is possible to trace rays in a 200 km x 200km model with an error of less than 10 ms. The error map at the 10th iteration shows no error patterns. A star-like error pattern is typical for ray tracing on regularly distributed grids.

azimuthally dependent. The algorithm was adapted to handle extreme topographic variations, and the resulting optimized network is shown, illustrating the self-densification properties of the algorithm. Future work includes extending the algorithm to 3D and using it as forward modelling for tomographic refraction statics.

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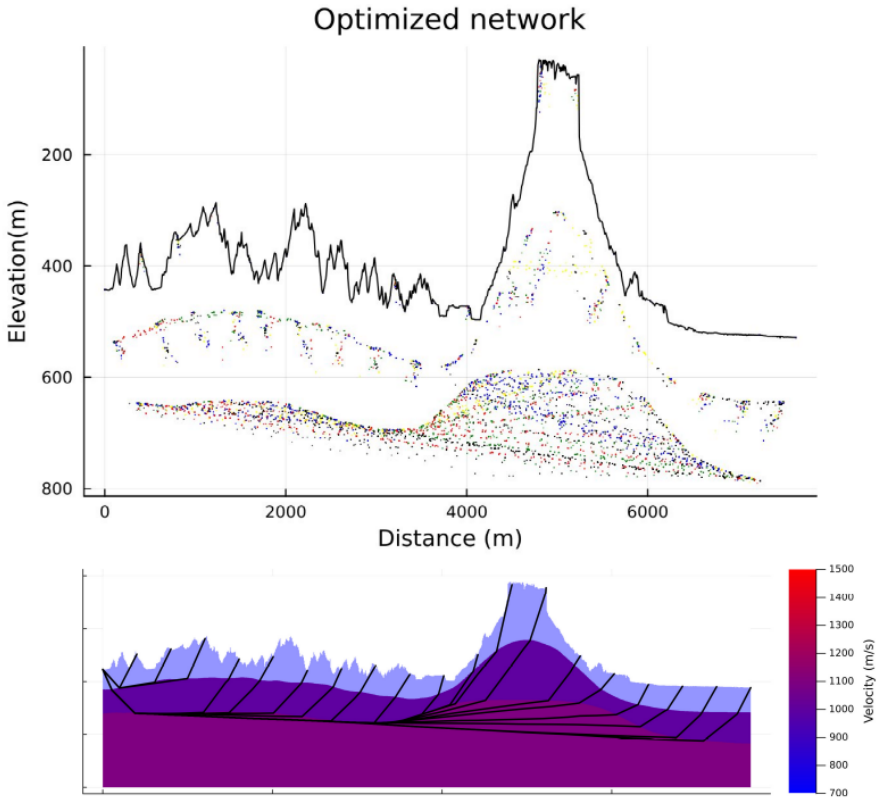

Figure 3: Top: Optimized network after five shots. Notice how the nodes required to trace rays map the high-impedance contrast surfaces. Bottom: The velocity model and the ray tracing correspond to one shot point.