

# Exploring the Possibility of Utilizing Seismic-While-Drilling and Full-waveform Inversion to Enhance Drilling Program Management

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## Summary

Managing drilling programs requires a comprehensive understanding of the subsurface geology. Full-waveform inversion (FWI) is an optimization-based approach that can aid in this understanding, but incomplete data and poor subsurface illumination can limit it. One possible solution to this problem is to include additional data from new and independent ray paths, which can be obtained using seismic-while-drilling (SWD) technology. The utilization of SWD can provide real-time information about the subsurface geology, thus allowing for better decision-making and reducing the risk of unexpected subsurface conditions. However, the inclusion of SWD data also introduces new unknowns such as the source's precise location and radiation characteristics that need to be considered in drilling program management. To address these challenges, we have developed a new elastic FWI algorithm that incorporates the unknowns of source position and radiation patterns, in addition to the velocity and density values of the grid cells. In this study, we conduct an experiment using synthetic data, which simulates a conventional acquisition geometry with the addition of SWD sources along a plausible well-trajectory. The well-recovered subsurface models and source unknowns indicate the impact of SWD on FWI, thus providing insight into how SWD can be used to enhance the accuracy of FWI models and aid in the management of drilling programs.

## Theory

### *Inverse problem*

The inverse problem in FWI is a nonlinear framework for minimizing the objective function (Tarantola, 1984). In the frequency domain, for a given frequency, the misfit function in the form of L2 norm is as follow:

$$\operatorname{argmin}_{\mathbf{m}} \phi(\mathbf{m}) = \frac{1}{2} \|\mathbf{R}\mathbf{u} - \mathbf{d}\|_2^2, \text{subjected to } \mathbf{S}\mathbf{u} = \mathbf{f} \quad (1)$$

where  $\mathbf{m}$  is subsurface model,  $\mathbf{R}$  is the sampling matrix which represents the measurements of receivers,  $\mathbf{u}$  stands for the wavefield from a source position to an arbitrary receiver,  $\mathbf{d}$  is the observed data between a source-receiver pair,  $\mathbf{S}$  is the finite-difference modeling operator, and  $\mathbf{f}$  is the source term. The calculation of objective function also includes the summation over frequencies (Pratt, 1990), but we do not state it here for convenience. When  $\phi(\mathbf{m})$  reaches the minimum value, the difference between synthetic and observed data is minimized, which means the current model is the most consistent with the recorded data. The optimization problem starts by building a second-order Taylor-Lagrange expansion of  $\phi(\mathbf{m})$ :

$$\phi(\mathbf{m} + \Delta\mathbf{m}) \approx \phi(\mathbf{m}) + \mathbf{g}\Delta\mathbf{m} + \frac{1}{2}\Delta\mathbf{m}^T \mathbf{H}\Delta\mathbf{m}, \quad (2)$$

where  $\phi(\mathbf{m})$ ,  $\mathbf{g}$  and  $\mathbf{H}$  are the objective function, gradient and the approximated Hessian matrix by Gauss-Newton method.

### Source representation

According to the Representative Theorem (Aki and Richards, 2002), the observed displacement can be represented by a set of dipole forces that cause the two blocks at opposite sides of the fault mutually move:

$$u_i(\mathbf{x}, t) = \int_{-inf}^{inf} M_{kl}(\tau) \frac{\partial}{\partial \xi_l} G_{ik}(\mathbf{x}, t; \xi, \tau) d\tau, \quad (3)$$

where  $u(\mathbf{x}, t)$  denotes the displacement at location  $\mathbf{x}$  and time  $t$ ,  $M_{kl}(\tau) = \iint_{\Sigma} m_{kl}(\tau) d\Sigma$  is the moment tensor defined along a fault surface  $\Sigma$  at time  $\tau$ ,  $m_{kl}$  is the moment tensor density,  $G_{ik}(\mathbf{x}, t; \xi, \tau)$  is the Green's function, and  $\xi$  is the location of drillbit source. Inspired by this insightful theory, we lump the bit part into a single oscillator coupled with the string, so the integral along the fault will be converted to an effective point on our 2D model. In our formulation, the horizontal and vertical components of the  $\mathbf{f}$  term in equation (1) are defined by:

$$\begin{cases} f_x = \sum_s \mathbf{w}(s)[M_{11}\omega_{dx}(s) + M_{12}\omega_{dz}(s)] \\ f_z = \sum_s \mathbf{w}(s)[M_{11}\omega_{dx}(s) + M_{22}\omega_{dz}(s)] \end{cases}, \quad (4)$$

where  $\mathbf{w}(s)$  is a spatial weighting matrix in which the entries are relevant to the skewing of a point source position and indexed by  $s$  denoting NE-SW-SE-NW.  $\omega_{dx}(s) = 1$  or  $-1$  when  $s$  denotes eastward or westward direction;  $\omega_{dz}(s) = 1$  or  $-1$  when  $s$  is southern or northern oriented, respectively. We provisionally consider the SWD sources as a combination of unknown random moment tensors. The detailed mechanism of drill-bit-rock interaction (Germa et al., 2009) is not yet described in our work. In contrast, we treat the surface sources as known explosive sources (isotropic).

### Example

The true and initial models are shown in Figure 1. A notable feature in the model is a high  $V_p$  and  $\rho$  anomaly which occurs at the same location as an obvious low abnormality. The model size is 300 by 150 grid points in horizontal and vertical directions, with a 20-meter interval. We consider the acquisition we are interested in characterizing shown in Figure 2. A deviation from the designated wellbore (yellow line) is added as the initial trajectory, as shown by the purple line.

This test assumes that the SWD sources are radiating independently from their positions. P-wave velocity, density, source radiations, and positions will be recovered. 10 frequency bands, each containing 12 sub-frequencies, are used in the multi-scale strategy. In every band, frequencies start from 1 Hz, while the ending frequencies linearly increase from 3 Hz to 20 Hz. Noise with different signal-to-noise (SNR) ratios are added separately to surface and SWD datasets.

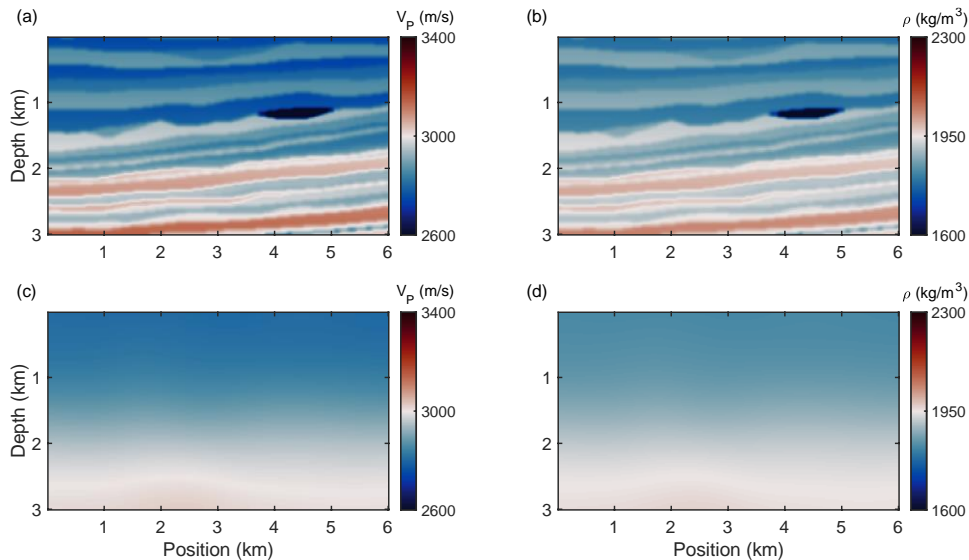


Figure 1. True and initial models. (a) true  $V_p$ . (b) true  $\rho$ . (c) initial  $V_p$ . (d) initial  $\rho$ .

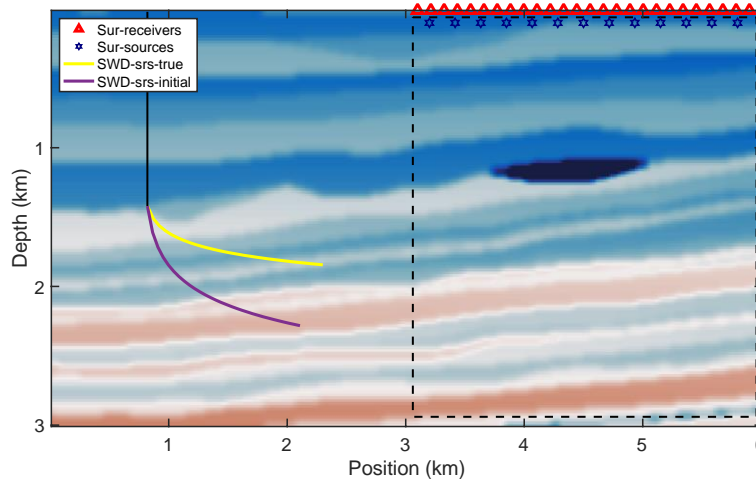


Figure 2. Acquisition system.

Figure 3 shows the inversion results of elastic properties with surface-only and noise-free settings as a baseline inversion. The depiction of both models could be more satisfactory. The error term is calculated with normalized root mean square error (N-RMSE). From Figure 4, which denotes the SWD-FWI inversion, a profound improvement can be observed, which indicates the essential role of SWD datasets in the FWI. Figure 5 illustrates a quite accurate recovery of moment tensors, as the initial values (blue circles) converged to true ones. The real drilling trajectories can also be depicted, as shown in Figure 6.

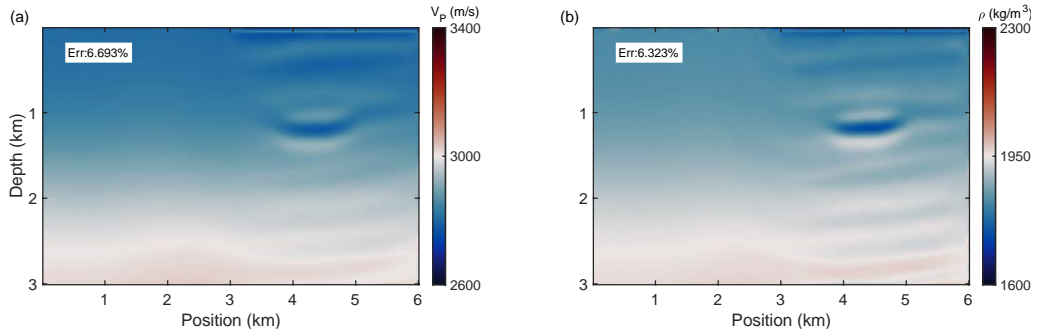


Figure 3. Baseline inversion. (a)  $V_p$  model. (b)  $\rho$  model.

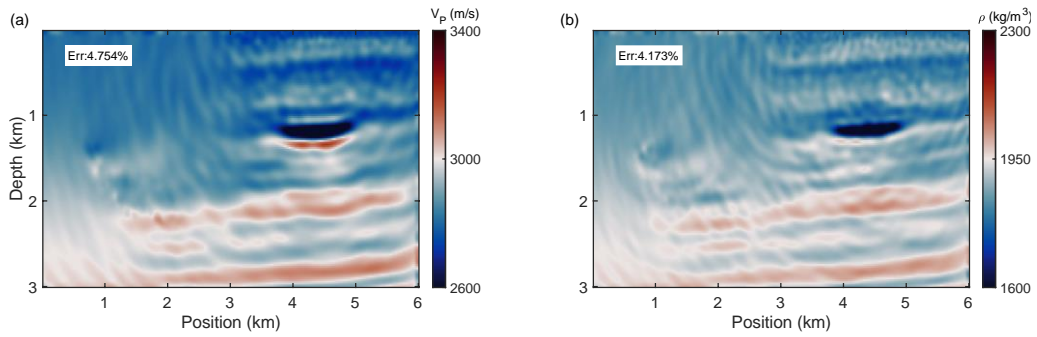


Figure 4. SWD-FWI inversion. (a)  $V_p$  model. (b)  $\rho$  model.

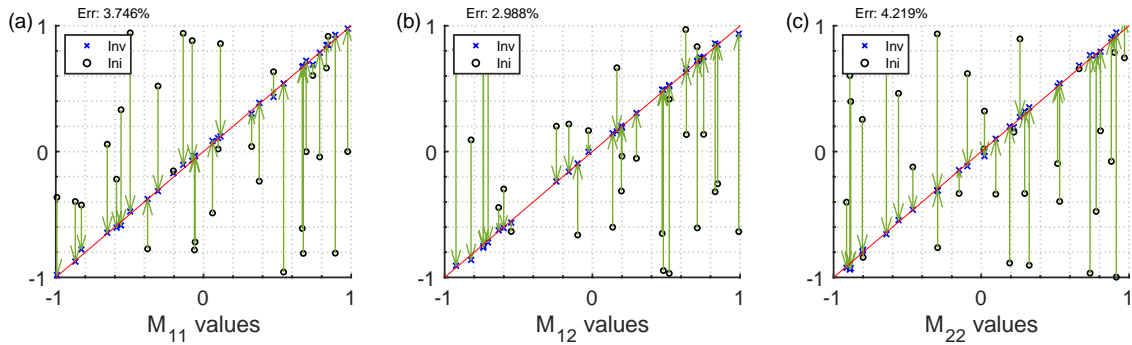


Figure 5. Cross-plots of moment tensor inversion. (a)  $M_{11}$ . (b)  $M_{12}$ . (c)  $M_{22}$

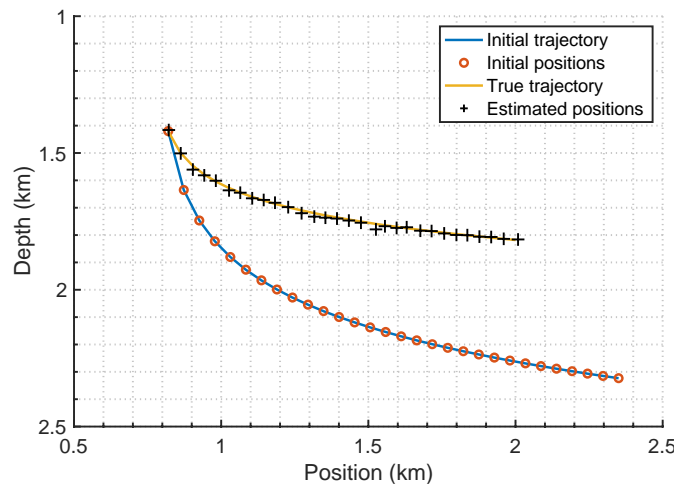


Figure 6. Drilling trajectory inversion.

## Conclusions

This study explored the potential of incorporating data from seismic-while-drilling (SWD) to improve the inversion results of compressional velocity, density, and moment tensors of unknown sources using full waveform inversion (FWI). The results indicate that utilizing SWD data can lead to considerable improvement in the inversion of subsurface properties. This study also shows that drilling-related source terms, such as radiation and deviation of trajectories, can be well-recovered as positive feedback from a better-estimated subsurface model, suggesting that incorporating SWD data into FWI can enhance the accuracy of subsurface models and aid in managing drilling programs. However, further research is needed to fully understand the mechanism of SWD sources represented by moment tensors and featured frequency spectrums, to draw more comprehensive conclusions about the impact of SWD on FWI and drilling program management.

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