

Linearized waveform inversion for vertical transversely isotropic elastic media

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Summary

Seismic migration and inversion are closely related techniques to portray subsurface images and identify hydrocarbon reservoirs. Seismic migration aims at obtaining structural images of subsurface geologic discontinuities. More specifically, seismic migration estimates the reflectivity function from seismic reflection data. On the other hand, seismic inversion quantitatively estimates the intrinsic rock properties of subsurface formulations. Usually, seismic migration and inversion techniques assume an acoustic or isotropic elastic medium. Unconventional reservoirs such as shale and tight sand formation have notable anisotropic property. We present a linearized waveform inversion (LWI) scheme for weakly anisotropic elastic media with vertical transversely isotropic (VTI) symmetry. It is based on two-way anisotropic elastic wave equation and simultaneously inverts for the localized perturbations from the long-wavelength reference model. The inversion has improved the quality of the images and reduces the multi-parameter crosstalk comparing with the adjoint-based images. We have observed that the multi-parameter crosstalk problem is more prominent in the inversion images for Thomsen anisotropy parameters. We also analyze the multi-parameter crosstalk using scattering radiation patterns.

Theory / Method / Workflow

We start the derivation of the method from a general forward problem for seismic waves in a VTI-elastic media. Under weak elastic anisotropy assumption (Thomsen, 1986), the elastic media can be described by the P- and S-wave velocities along symmetric axis plus Thomsen anisotropic parameters. The propagation of seismic wave satisfies the equations of motion

$$\mathcal{A}\mathbf{u} = \mathbf{f},$$

where \mathcal{A} is elastic wave operator, \mathbf{u} is seismic wavefield and \mathbf{f} is seismic source. The seismic inverse problem entails inferring subsurface model parameters from the seismic data recorded at the earth surface. In mathematical jargon, the seismic inverse problem is to optimize the coefficients of the partial differential equations (PDEs) from the boundary measurements. This is the well-known full-waveform inversion (FWI) problem in geophysics community. High computational cost, local minima and multi-parameter crosstalk are among the main challenges for FWI. A less ambitious approach is the linearized waveform inversion (or Born inversion) that replaces the nonlinear wave equation modeling operator by the linearized Born approximation modeling operator (Chen and Sacchi, 2017) and solves for localized perturbations with respect to the reference model

$$\mathcal{A}^0\Delta\mathbf{u} = -\Delta\mathcal{A}\mathbf{u}^0,$$

where \mathcal{A}^0 is elastic wave operator in background model, \mathbf{u}^0 is wavefield in background model, $\Delta\mathbf{u}$ is scattered wavefield and $\Delta\mathcal{A}$ is the scattering potential. The inversion is to find the model vector from the seismic reflection data. However, the inverse problem is often ill-posed because of the imperfect sampling of the seismic acquisition geometry. Besides, the observations are

usually contaminated with noise that cannot be explained by the forward operator. We pose the following linearized waveform inversion for VTI-elastic media as

$$\text{minimize } \chi(\Delta\mathbf{m}) := \|\mathcal{W}_d(\mathcal{L}\Delta\mathbf{m} - \Delta\mathbf{d}^{\text{obs}})\|_2^2 + \mu\|\mathcal{W}_m\Delta\mathbf{m}\|_2^2,$$

where \mathcal{L} is the linearized forward modeling operator, $\Delta\mathbf{d}^{\text{obs}}$ represents the multicomponent reflection data, $\Delta\mathbf{m}$ denotes model vector, $\|\cdot\|_2^2$ represents the L_2 norm of vector, μ is the trade-off parameter, \mathcal{W}_d and \mathcal{W}_m are a priori weighting operators for data and model, respectively. We use the conjugate gradient method to solve the least-squares optimization problem. We derive the expressions of radiation patterns of VTI-elastic scatters using the Born approximation and high-frequency, far-field asymptotic Green's function (Podgornova et al., 2018). The scattering function for an arbitrarily anisotropic scatter embedded in a homogeneous anisotropic background model is

$$R = \Delta\rho V\tilde{V}g_i\tilde{g}_i + \Delta c_{ijkl}g_k\tilde{g}_i p_l\tilde{p}_j,$$

where $\Delta\rho$ and Δc_{ijkl} are model perturbation, V is the phase velocity, g_i denotes polarization vector and p_l represents the unit slowness vector of the incident wavefield. The tilde (\sim) superscript is used to indicate quantities associated with scattered wavefield (receiver-side).

Results, Observations, Conclusions

As a first example, we consider a layer-inclusion VTI-elastic model. This example illustrates the performance of LWI for multi-parameter decoupling. Fig. 1 shows the inversion results obtained by our proposed VTI-elastic LWI. The iterative inversion reduces the low-frequency artifacts and acquisition footprint in the images. Also, the spatial resolution of the images is improved. More important, the VTI-elastic LWI attenuates the multi-parameter crosstalk through iterative data fitting. The results by LWI are not perfect due to the ill-conditioned nature of the inverse problem.

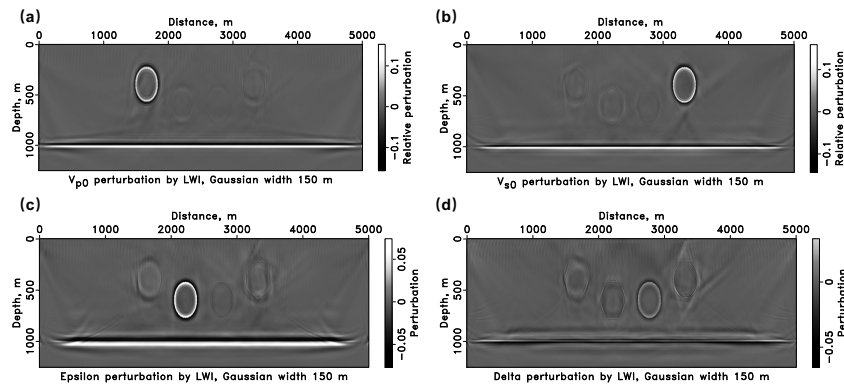


Fig. 1 Model perturbations estimated by proposed VTI-elastic LWI. (a) Relative perturbation of P-wave vertical velocity $\Delta V_{p0}/V_{p0}$. (b) Relative perturbation of S-wave vertical velocity $\Delta V_{s0}/V_{s0}$. (c) Perturbation of Thomsen parameter $\Delta\varepsilon$. (d) Perturbation of Thomsen parameter $\Delta\delta$.

The isotropic parameters (V_{p0} and V_{s0}) are better resolved than the anisotropic parameters (ϵ and δ). The Thomsen anisotropy parameters are more prone to multi-parameter coupling. Especially, the parameter δ is the most difficult parameter to recover. The reason for this phenomenon is that Thomsen parameter δ has less impact on the reflection seismic data comparing with other elastic parameters.

For a more complicated example, we next consider a modified version of the VTI-elastic Hess model. The results of inversion-based VTI-elastic LWI are shown in Fig. 2. The inversion improves the horizontal and vertical spatial resolution throughout the images. It also corrects the geometric spreading effects that boosting up the image amplitude in the deeper part. Besides, the linearized inversion remarkably attenuates low-frequency artifacts in images via iterative data fitting.

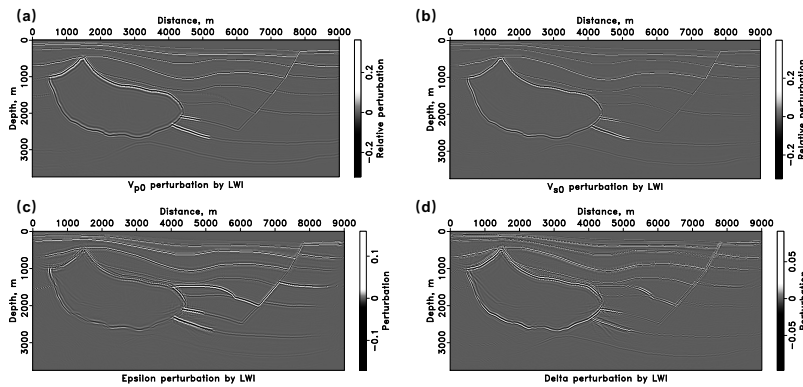


Fig. 2 Model perturbations estimated by proposed VTI-elastic LWI. (a) Relative perturbation of P-wave vertical velocity $\Delta V_{p0}/V_{p0}$. (b) Relative perturbation of S-wave vertical velocity $\Delta V_{s0}/V_{s0}$. (c) Perturbation of Thomsen parameter $\Delta\epsilon$. (d) Perturbation of Thomsen parameter $\Delta\delta$.

In Fig. 3, we plot the scattering radiation patterns of perturbations of VTI-elastic parameters ($\Delta\rho/\rho, \Delta V_{p0}/V_{p0}, \Delta V_{s0}/V_{s0}, \Delta\epsilon, \Delta\delta$) in isotropic elastic reference medium. The scattering radiation patterns are plotted in polar coordinates as a function of the scattering angle (open angle between incident and scattered wave). Scattering radiation patterns for different elastic parameters overlap and cause the so-called crosstalk artifacts in waveform inversion. The scattering radiation patterns of Thomsen anisotropy parameters have smaller amplitude comparing with the ones of vertical velocities. The Thomsen anisotropy parameters are mainly sensitive to intermediate to large scattering angles.

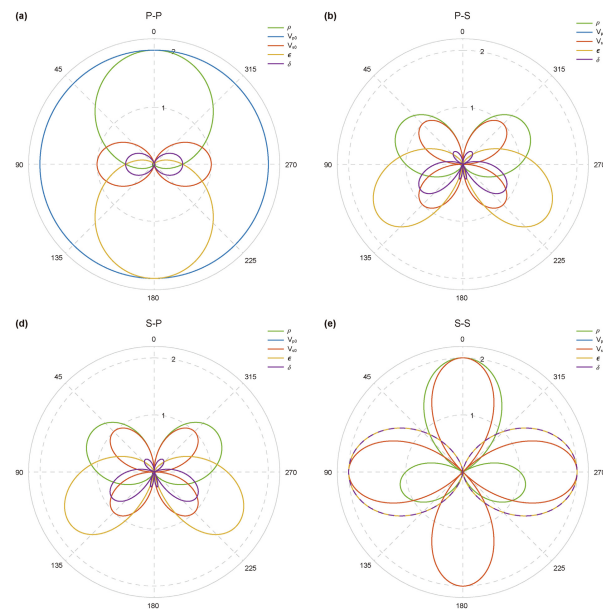


Fig. 3 Radiation patterns for VTI-elastic scatters. (a) P-P radiation patterns, (b) P-S radiation patterns, (c) S-P radiation patterns, (d) S-S radiation patterns.

Conclusions

We study a linearized waveform inversion method for VTI-elastic media and analyze the multi-parameter crosstalk utilizing scattering radiation patterns. The proposed scheme simultaneously inverts for the localized model perturbations with a prior knowledge of the long-wavelength reference model. The linearized inversion improves the spatial resolution, reduces image artifacts and multi-parameter crosstalk and improves amplitude response of the elastic images comparing with the adjoint-based images. We have observed that the multi-parameter crosstalk problem is more prominent in the inversion images for Thomsen anisotropy parameters. The observations are supported by analyzing the scattering radiation patterns.

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