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On the stability of stress inversions from earthquake mechanisms

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Summary

Recent work using stress inversions has emphasized the lack of stability of stress inversions from earthquakes with fault planes of limited diversity. We explore this question using data recorded over an enhanced monitoring array and address how we can answer questions on stability. Caution must be used with stress inversions to use knowledge of the fault plane in such cases, a false impression of stability may be offered running these algorithms using random choices of nodal planes.

Introduction

Lundstern et al. (2024) conducted a synthetic experiment with an eye to the public catalog of seismicity in the Midland Basin of Texas to illustrate the criticality of having a diverse set of mechanisms on the resolution of stress inversions. This area makes for a good natural laboratory for such a test, as there are numerous faults in this area with consistent mechanisms across the area. The authors restrict their analysis to the case where the usual problem of identifying the actual fault plane from the auxiliary plane has already been done. They recommend that an examination of the posterior covariance matrix of the inversion can yield fruitful insights on the reliability of the inversion, and examine the null-space of the inversions to highlight some of the issues. Because their examples are synthetic, they have ground truth and can directly compare resolved stress to the known input stress.

In this paper, we examine the catalog of seismicity from this basin, where we use data from a private seismic array combined with the public array to investigate where stress inversions may or may not be well-resolved. Our catalog of events is real, and we run stress inversions over mechanisms for events greater than M2. We do observe that an overly-consistent mechanism distribution has deleterious effects on stress resolution, though a Monte Carlo analysis. Critically we see how running stress inversions agnostic to the choice of fault plane (in other words, randomly picking the fault plane) can create the illusion of stability. In addition to examining the inversion matrix structure, we show how the stability of the inversions may be visualized.

Theory

The problem of stress inversions is well-examined and interestingly has received attention over the past 50+ years. By invoking the Wallace-Bott assumption that faults slip in the direction of maximum shear stress, a system of equations can be constructed in terms of the geometry of the fault, the normalized deviatoric stress, and the slip vectors (Michael, 1984):

$$\mathbf{\Sigma}\hat{\mathbf{n}}_i - (\mathbf{\Sigma}\hat{\mathbf{n}}_i \cdot \hat{\mathbf{n}}_i)\hat{\mathbf{n}}_i = \tau_i\hat{\mathbf{s}}_i$$

In the equation above, the stress tensor, $\mathbf{\Sigma}$, appears on the left-hand side and the right-hand side through traction, τ. The fault geometry appears in the unit normal vector, $\hat{\boldsymbol{n}}$, and the unit slip vector is given as $\hat{s}$. Micheal (1984) constructed a linear system of equations by assuming that shear stress was constant on the all faults which had the effect of dropping the stress parameters on the right-hand side of the equation. Beaucé et al (2022) introduced a modification to this strategy to allow for variable shear stresses by staggering the linear system in with steps to recompute the traction and it is their algorithm we use in our analysis.

Moment tensors (almost) yield all the geometrical information necessary for this system, save for the equivalence of the nodal planes (in the double-couple approximation) with one of the fault planes means that there is a degeneracy that needs to be addressed. Many inversion algorithms offer the ability to resolve this ambiguity, but the problem is highly non-linear since the choice of plane constructs a different inversion matrix.

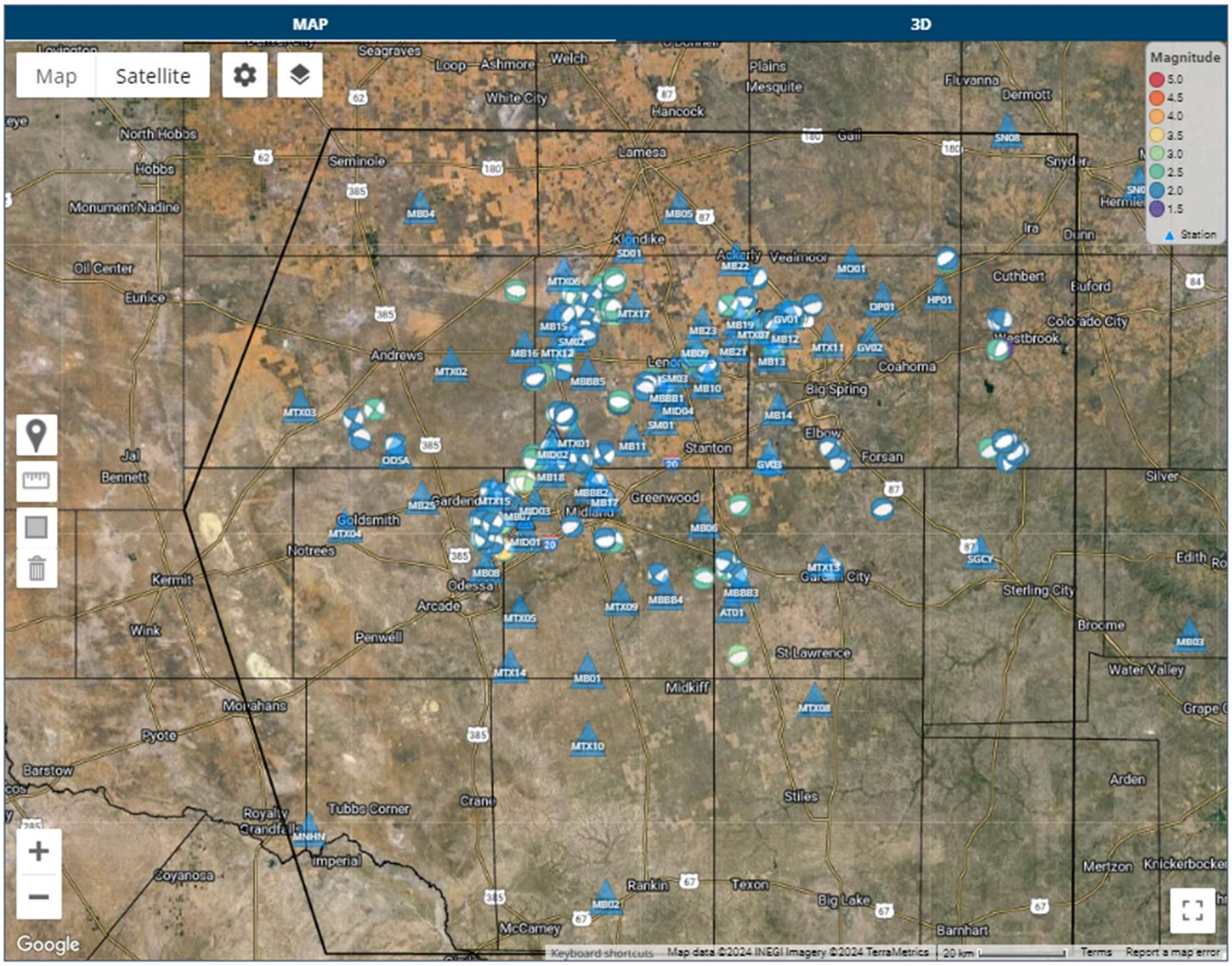

Figure 1. The Midland Basin network (triangles) and moment tensors for events up to 2024

Dataset

Since 2020, Nanometrics has been operating a private network of seismometers in the Midland Basin around Midland and Odessa, Texas, and surrounding counties. This network complements the public network of stations in that basin, similar to that which exists in the Delaware Basin described by Baig et al. (2022). Since April 2020, over 12000 events have been detected and located (in the Midland Basin) complete over the main footprint of the array to below M_L1. For the vast majority of over 400 events above M_L2 in the array, we determine moment tensors through either an amplitude-based inversion or failing that a polarity-based inversion of P, SV, and SH phases in a scheme described in Baig et al. 2022. Mechanisms are normal to strike-slip, varying across the region, and are consistent with a tensional stress axis that is NW to N. Figure 1 shows these mechanisms as beachballs against the array.

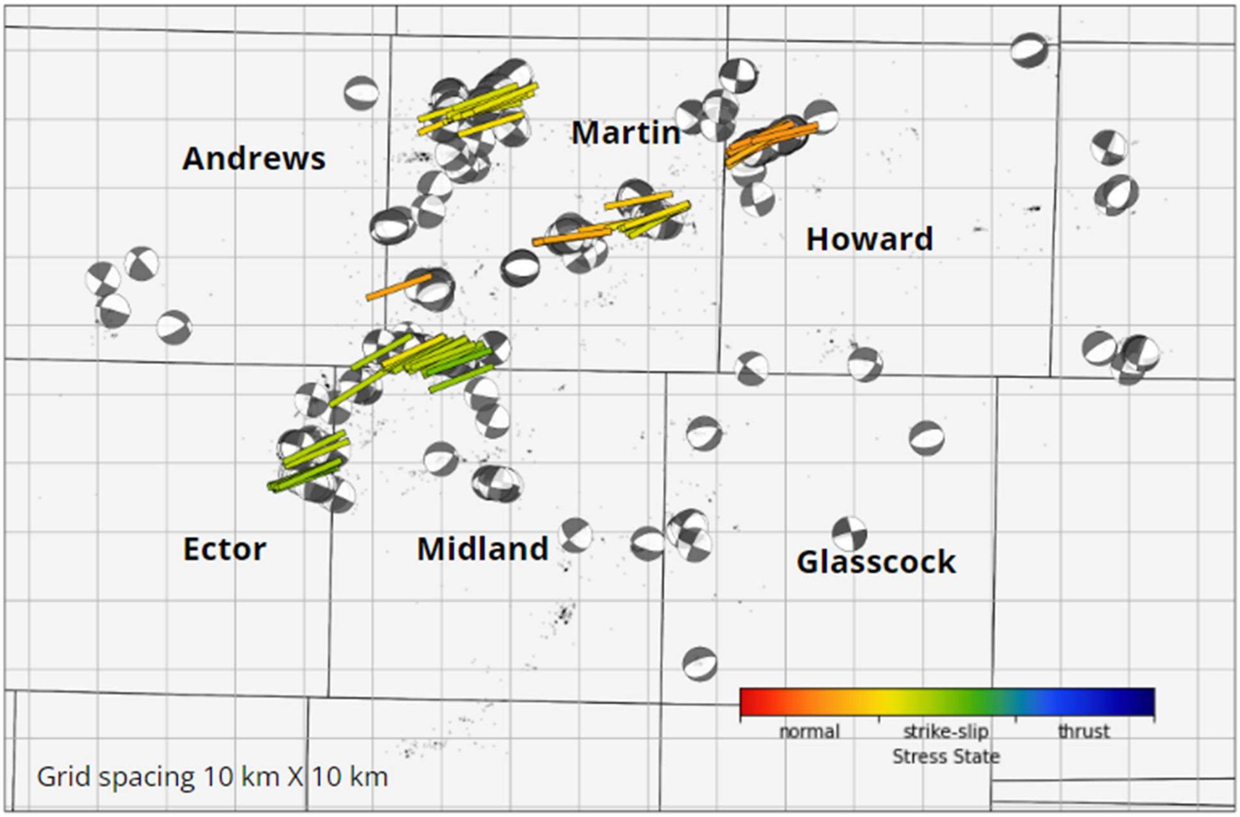

Figure 2. An example spatial stress inversion over the Midland basin agnostic of fracture plane choice. Certain counties are noted and the description of the tick marks indicating stress is given in the text.

Methodology

We have been running stress inversions regularly against this dataset, using Beaucé et al.'s 2022 algorithm in the mode where we did not choose a fracture plane. Parameters of this inversion are identical to those described in Baig et al. (2022), notably that we use nearest-neighbor groupings of mechanisms with a minimum of 20 events with a 10 km radius. Results of the inversion are realized as vectors oriented along the resolved SHmax direction and colors by Simpson's 1997 A_φ parameter. While we have done a cursory examination of mechanism variety in these nearest neighbor clusters, along the suggestions of Hardebeck and Hauksson, 2001, we do a deeper dive into mechanism stability along the lines suggested by Lundstern et al. 2024.

Lundstern points out that a lack of stability in the inversion can manifest itself in the posterior covariance matrix of the inversion, following the inversion formalizing presented by Tarantola and Villotte, 1982. Where there are no significant tradeoffs between resolved parameters in the inversion, this matrix is close to the Identity matrix after normalization. Significant off-diagonal terms, on the other hand, reveal instability. Inversions only using events from a limited set of fault orientations they show lead to unstable and systematic errors in the resolved stress state. While we cannot replicate their methodology with real data examples, we can use their insights into our dataset, as the choice of fault plane in many circumstances in this dataset is rather trivial as high-precision locations for strike-slip faults very strongly indicates one possible plane over another. For this reason, we focus our analysis on clusters of events in the Gardendale area of the study area, towards the SW of the map (see Figure 3).

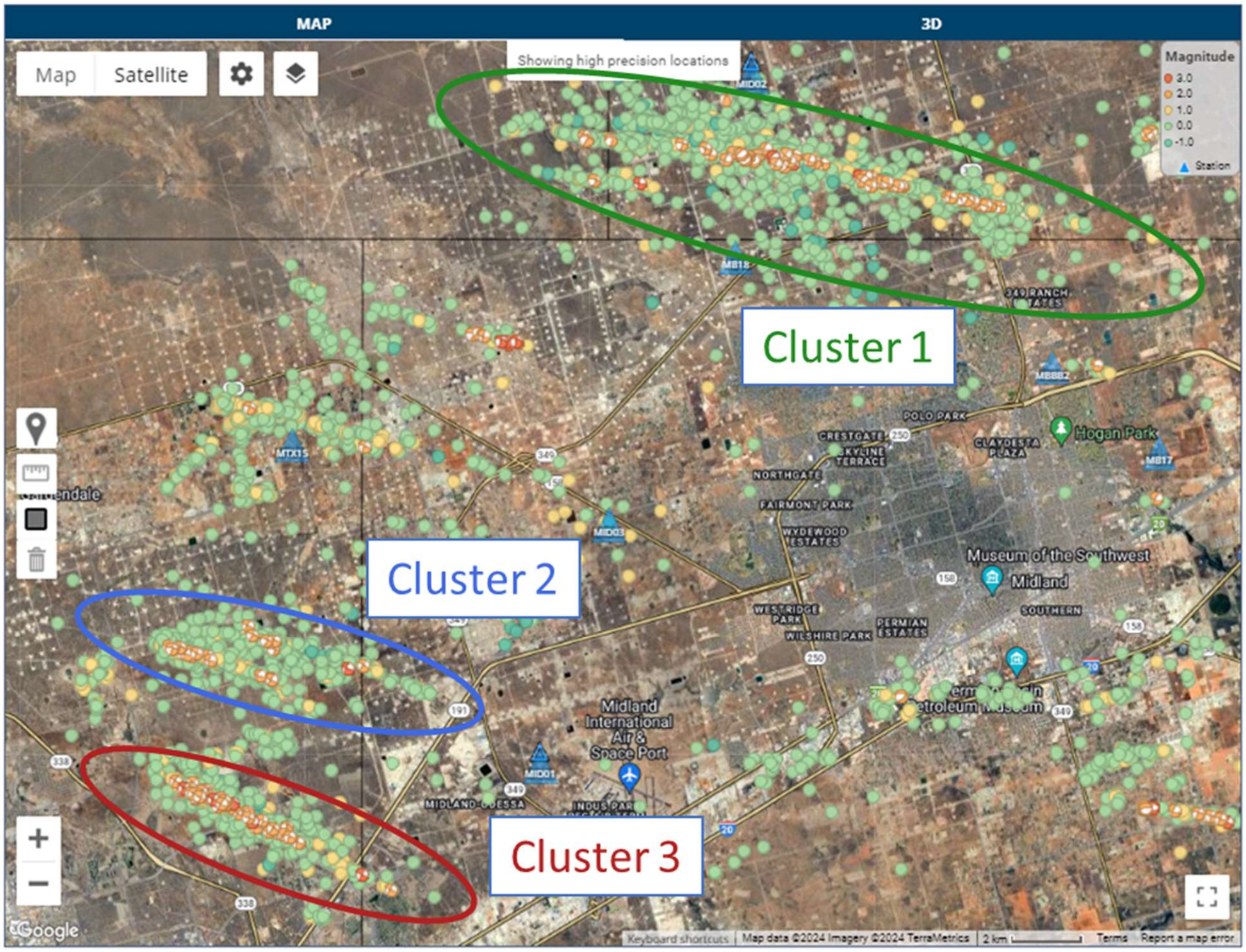

Figure 3. High precision locations of seismicity around the city of Midland in the SW of the array. We identify three clusters for further analysis.

We propose in addition to looking at this matrix, a more direct observation of the inversion instability is available through a simple Monte Carlo analysis. Perturbing the input mechanisms within reasonable error bounds can immediately highlight the tradeoffs in resolved stress parameters. In our analysis we perturb the normal vectors by a randomly by a rotation with a standard deviation of 10° and the rakes by an angle drawn from a Gaussian distribution of also 10°. We display the resultant 100 iterations of the variable stress inversion to show the inversion tradeoffs and run the inversions with fault planes chosen to align with high-precision clusters of lineations as well as randomly choosing nodal planes as fault planes.

The southern clusters (cluster 2 and 3) have very consistent orientations all more-or-less aligned along the fault planes highlighted by the high-precision locations. For the northern-most cluster (1) the majority of the events are strike slip with a fault plane that is more or less obvious to pick. For the few normal mechanisms in the dataset, we choose one of the fault planes arbitrarily. A detailed three-dimensional analysis may help disambiguate these planes, but for the purposes of this study, we just note that these mechanisms, potentially highlighting a small pull-apart basin provide a good source of fault plane diversity.

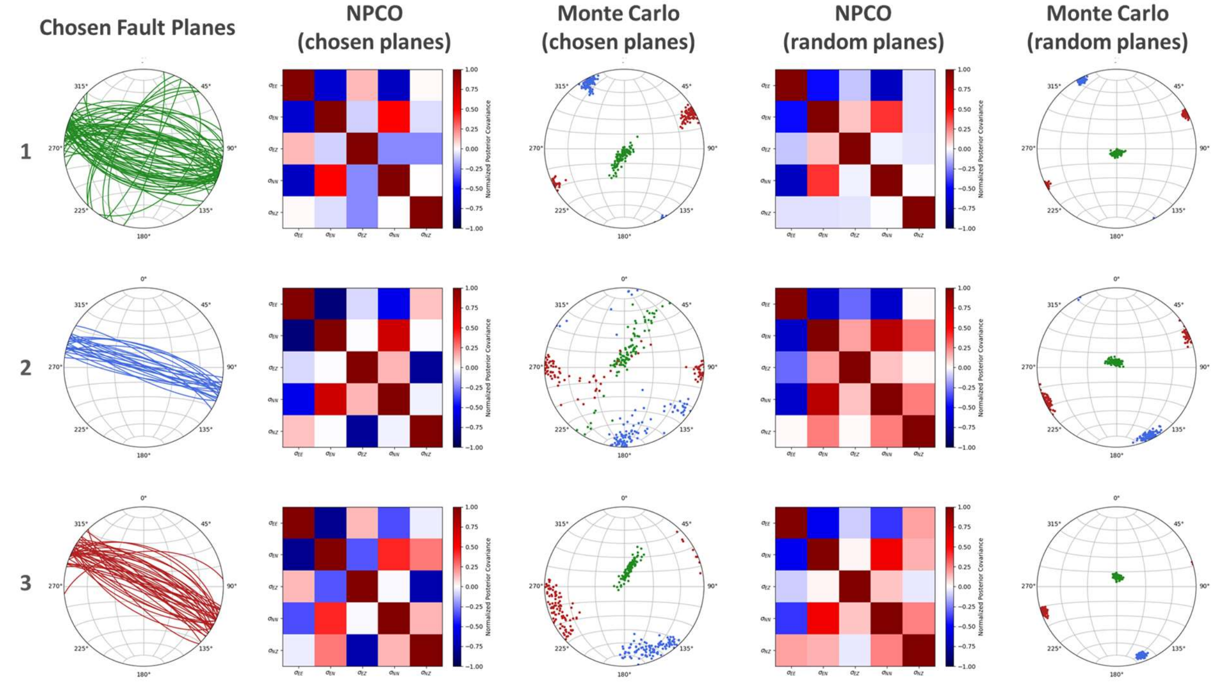

Figure 4 The row correspond to the three clusters highlighted in Figure 3. The chosen fault planes (left); the normalized posterior covariance matrixes and distribution of stress axes from the Monte Carlo analysis for the inversions using the selected fault planes (columns 2 and 3); and the same as columns 2 and 3 for the last pair of columns where fault planes are randomly chosen.

In figure 4, we show both the input chosen fault orientations as well as the normalized posterior covariance (NPCO) matrixes (as suggested by Lundstern et al., 2024) and the distributions of resolved stress axes from running the stress inversions in the Monte Carlo fashion as described above. The NPCO matrixes and stress axes are displayed for both the algorithm run with the chosen fault planes (columns 2 and 3) and with random nodal planes used as input. (columns 4 and 5). The choice of random nodal plane does not vary between Monte Carlo iterations for this latter set. Note that these NPCO matrixes are 5x5 because we resolve the deviatoric stress tensor, so the apparently missing σ_{ZZ} term is elided because is it completely determined by the requirement that the deviatoric tensor have zero trace. Although the stress tensor shape ratio completes the deviatoric tensor that we observe, to focus the discussion we do not show it.

Focusing on the stress inversions with the chosen fault planes, the NPCO matrixes for clusters 2 and 3 show some very strong tradeoffs, with values in the off-diagonal terms approaching values of –1 showing strong tradeoffs between σ_{EE} and σ_{EN} as well as between σ_{EZ} and σ_{NZ}. These instabilities are perhaps more easily visualized in the Monte Carlo analysis. Starting with cluster 3, the most compressive and least compressive axes (red and blue, respectively) show very strong scatter indicating that resolved SH_{max} directions can span a range of 45°. This range is perhaps unsurprising from basic mechanical considerations of the range of possible stresses that can cause a single fault to slip. Cluster 2 shows a similar range, although the stress axes are distributed bimodally. The intermediate (green) axis is consistent with vertical in both cases, but shows significant scatter Cluster 3, with its sub-population of normal faults, shows more stability. The NPCO matrix does not have nearly so strong off-diagonal terms (although still significant) and the Monte-Carlo result shows a higher degree of stability when compared with the other two clusters.

Shifting attention to the random nodal plane examples, we observe more stability in these inversions: off-diagonal terms in the NPCO matrixes are relatively attenuated from the chosen planes and the Monte Carlo stress axes form tight concentrations on the stereonets. This apparent stability is a complete illusion. The incorrect selection of auxiliary planes simulates a situation with strong fault plane diversity that does not exist. As such we restate a note of caution that others have likely stated before: that in cases with low mechanism diversity selecting the nodal plane that corresponds to the actual fault plane is critical to retrieving accurate stress inversions, and that one must be wary of the precision offered by avoiding this choice.

Discussion

The modest amount of examples presented in this paper (we think) very enlightening in terms of how to operate stress inversions in general. In cases where there is not a great diversity of fracture planes, stress inversions are poorly resolved as has been claimed and shown numerously before now. In such cases, it is imperative that one not run stress inversions agnostic of which nodal plane is the fault plane (perhaps this comment is restricted to the family of algorithms descending from Michael's 1984 work). Either the fracture plane can be deduced from geological information, from hypocenters, or potentially the fault plane can be selected in the course of the inversion (Vavryčuk, 2014 and Beaucé et al. 2022 both offer this capability). Ignoring this warning may give a false sense of confidence in the inverted results.

With sufficient diversity in mechanisms, it is an open question whether running a stress inversion can be run in "agnostic" mode and we intend to look into this in the future. Furthermore, we need to investigate how to properly assess mechanism diversity. Hardebeck and Hauksson (2001) and Lundstern et al. (2024) recommend looking at the distribution of rotations in the dataset away from the average mechanism. We suspect that this recommendation can be quantified in terms of the structure of the inversion matrix.

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Beaucé's dedication to giving worked examples of his stress inversion including diving into the stability of the inversions on his Github page.

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