

Elastic and Thermal Cross-Property Relationships in Porous Media: An Inverse Modelling Approach

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Summary

Thermal conductivity plays a crucial role in optimizing thermal processes in industries, particularly in geothermal operations. Direct measurement of thermal conductivity presents various challenges at both field and laboratory scales, making accurate estimations based on other properties highly valuable. This study evaluated the thermal conductivity of various porous media models, exploring its relationship with fundamental frequency (for laboratory measurements) and other elastic properties such as Bulk and Young's Modulus (for field measurements) through a frequency domain study. Finally, the P-wave modulus was chosen as the most correlated elastic parameter with thermal conductivity and corresponding relationships was suggested in different saturation conditions.

Theory / Method / Workflow

Thermal conductivity is a key property affecting heat transfer in materials, critical in fields like science and engineering, particularly in optimizing energy processes like geothermal operations[1]. This property is highly influenced by factors like porosity and mineralogy. Traditional methods to measure thermal conductivity, divided into steady-state and unsteady-state, often face challenges like time consumption, inaccuracies, and sample damage. In addition, mentioned approaches are typically developed for laboratory measurements rather than field measurements[2, 3]. In the recent decades, many researchers have tried to establish cross property correlations to assess thermal conductivity of porous materials based on their elastic properties through the laboratory data. However, results from these studies are not confirming each other, and in some cases, weak correlations are suggested and in other cases no clear relationship was reported. In addition, most of studies have been done for dry samples and all of the available literature works are based on P-wave velocity and there is no frequency-based study nor other elastic parameters[4-7]. In this study, first the traditional porosity-based relationships for prediction of thermal conductivity were assessed and then cross property correlations were suggested based on the fundamental frequency for lab applications and based on elastic properties with the aim of its application in in-situ measurements.

In the initial phase of the simulation, a variety of 2D and 3D geometries, including 4x4 mm squares and 4x4x4 mm cubes, were generated, each featuring diverse porosity levels and pore configurations. For each model, porosity and permeability were evaluated to gain insights into the petrophysical characteristics. Subsequently, the thermal conductivity of each model was determined by applying the Fourier Heat Law in all three dimensions at the pore level. This involved an inverse modeling technique to align the results from macroscopic and microscopic simulations, using the temperature gradient across the material as the key parameter for matching. Initially, granite (G) and water (W) were chosen to represent the solid and liquid phases, respectively, under the assumption of a fully saturated medium. However, as the research progressed, further tests incorporated silica glass (S) as the solid phase and air (A) as the liquid phase, resulting in four distinct material combinations: GW, GA, SW, SA. Following the determination of thermal conductivity for each case, the models were then subjected to an evaluation of their elastic properties. The first step in this assessment involved determining each

model's fundamental resonant frequency through a frequency domain analysis. This analysis integrated specific equations and defined appropriate boundary conditions across different physical domains to obtain the fundamental resonant frequencies. Subsequently, the Young's modulus (E) and Bulk modulus (K) were calculated for all models using another frequency domain approach that emulated actual laboratory procedures, relying on stress and strain measurements. Ultimately, with the values of E, K, and the densities of the media at hand, additional elastic properties such as P and S wave velocities, velocity ratio, Poisson's ratio, Shear modulus, the first Lamé constant, and the P-wave modulus were calculated based on their well-established mathematical relationships.

Results, Observations, Conclusions

As it Shown in Fig.1 thermal conductivity of all four cases (GW, GA, SW, SA) indicates obvious deviations from typical geometric and parallel models. By calculation of Mean Squared Error (MSE) for all these cases (Fig.2) it is obvious that as the difference in thermal conductivity of two present phase increases, deviation values start to increase. This inaccuracy in prediction of thermal conductivity by porosity values underscore the importance of developing different measures for thermal conductivity predictions.

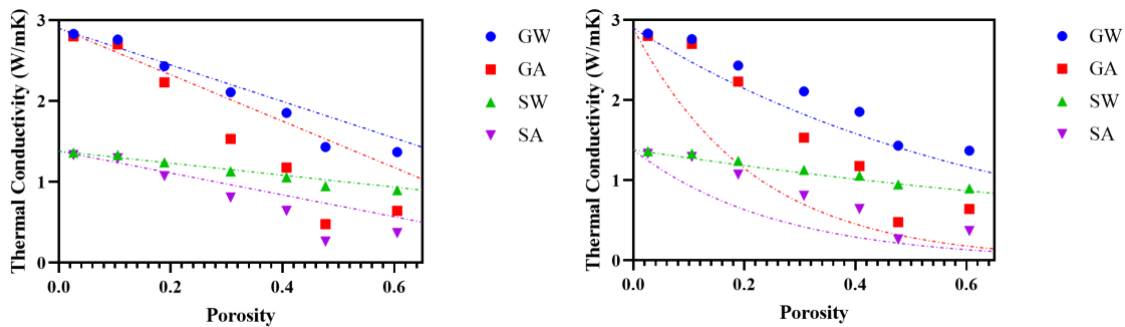


Figure 1- Comparison of Parallel Model (left) and Geometric Model (right) Fit Quality for In All Four Available Material Schemas

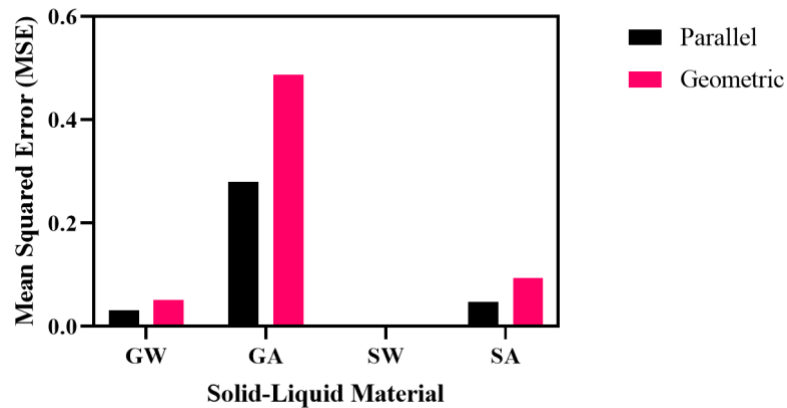


Figure 2- Mean Squared Error in Each Solid-Liquid Pair for Parallel and Geometric Model

Plotting thermal conductivity against fundamental frequency for each model, under both saturated and dry conditions, reveals a pronounced inverse linear relationship between these parameters. Employing a straightforward pulser-receiver setup in a laboratory setting allows for the rapid assessment of a sample's thermal conductivity, within seconds, using a non-destructive technique. Notably, the gradient of thermal conductivity declining behaviour in dry conditions is higher than in saturated scenarios.

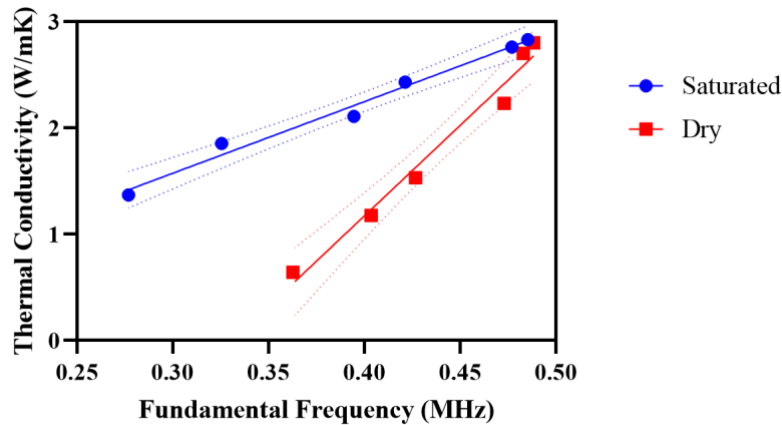


Figure 3- Thermal Conductivity of Granite as a Function of Fundamental Frequency in Both Dry and Saturated Cases

Following the computation of Bulk and Young's moduli (Fig 4), additional elastic properties were determined. As other studies mainly investigated the P-wave velocity relationships with thermal conductivity, calculation of this parameter, led into a relatively strong linear relationship (Fig 5). Results showed that in saturated cases, this relationship is a positive relationship, however in dry cases P-wave velocity decrease as the thermal conductivity rises. An evaluation of the Pearson correlation between thermal conductivity and the measured elastic properties revealed that the P-wave modulus exhibits the strongest correlation with thermal conductivity (Fig.6).

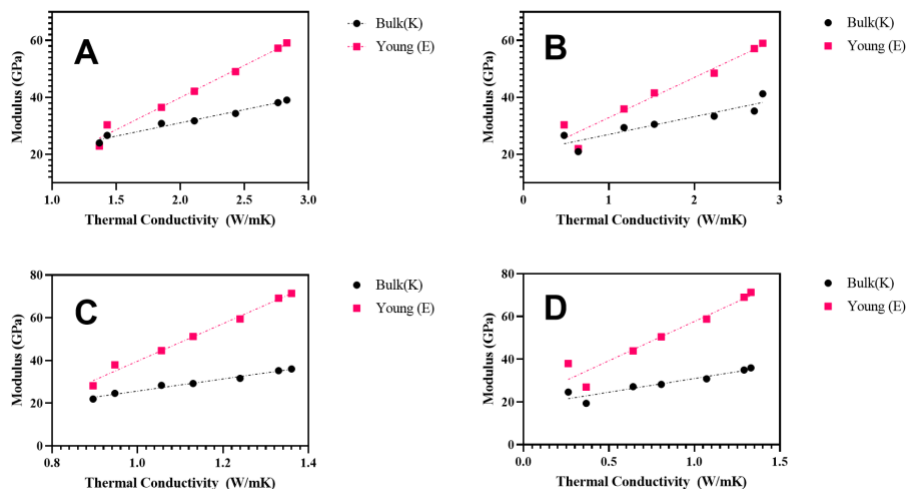


Figure 4- Bulk Modulus (K) and Young Modulus (E) in All Four Solid-Liquid Pairs (A:GW, B:GA, C: SW, D:SA) as a Function of Thermal Conductivity

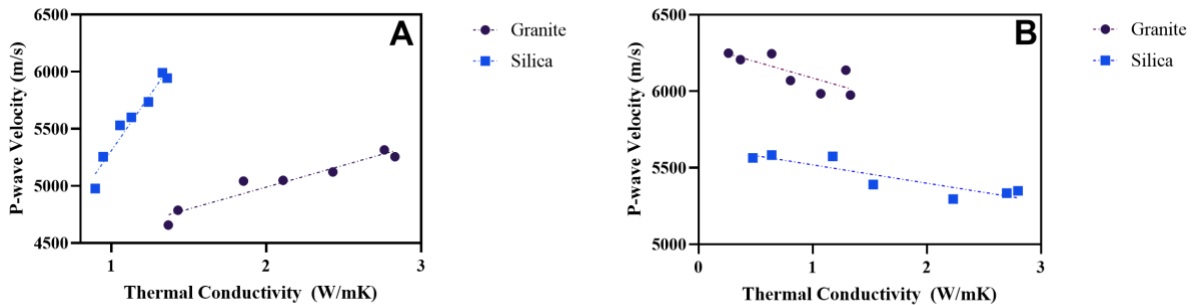


Figure 5- P-wave Velocity in Both Saturated (A) and Dry (B) cases.

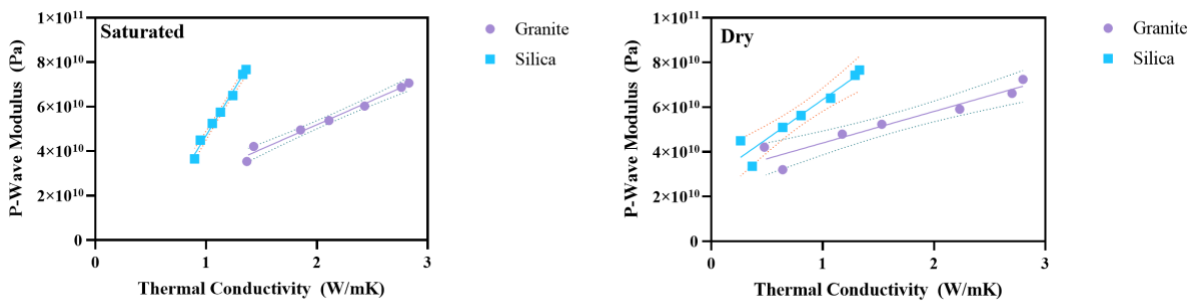


Figure 6- P-wave Modulus as a Function of Thermal Conductivity in Both Dry and Saturated Cases

Novel/Additive Information

Upon examining the insights presented in the Results and Conclusion sections, the following novel findings and conclusions are delineated:

- Conventional models for predicting thermal conductivity, such as parallel and geometric models that consider porosity, fall short in capturing the full range of pore structures and material diversity.
- The discrepancy between the calculated thermal conductivity and the values predicted by geometric and parallel models diminishes as the thermal conductivity contrast between the solid and liquid phases increases. These models yield more accurate results when the thermal conductivities of the two phases are similar.
- A linear correlation has been established between thermal conductivity and fundamental frequency in both dry and saturated conditions. This relationship facilitates the prediction of thermal conductivity in laboratory settings using a rapid, non-destructive method.

- A significant correlation was observed between thermal conductivity and both Bulk and Young's Moduli. However, P-wave velocity showed different trend in dry and saturated cases versus thermal conductivity. Since elastic properties are key parameters obtained from well-logging operations, estimating thermal conductivity based on these values could be advantageous, leveraging existing log data.
- The P-wave Modulus exhibited the strongest Pearson correlation with thermal conductivity, suggesting its potential as the most reliable predictive tool for estimating thermal conductivity based on elastic properties.

Acknowledgements

Financial support from the Energi Simulation Centre for Geothermal Systems Research, The Energy Harvesting Processes Program (ConocoPhillips, Ashaw Energy, Kalina Distributed Power, Telsec, Remedy Services, The Alberta Geological Survey and The Geological Survey of Canada) and the Fundamentals of Unconventional Resources program (NSERC, Alberta Innovates, Chevron, CNRL, ConocoPhillips and Enerplus) is gratefully acknowledged.

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