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# Geometrical and Electrical Tortuosity in Virtual Petrophysical Laboratory

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## Summary

An important characteristic of porous media is the geometrical connectedness of the porous network which can support the movement of fluids. The concepts, simulated processes, and diagnostics supported by the Virtual Petrophysical Laboratory (VPL) were presented in the first four papers[1,2,3,4] at Geoconvention in 2020, 2021, 2022 and 2024. During 2024 Geoconvention[5] we presented Archie's 'm' and 'n' exponents derived for virtual samples created and tested in the VPL. That was accompanied by electrical tortuosity estimates for all samples and two separate values for samples with the homogeneous and correlated pore networks.

This paper presents a novel process for geometrical tortuosity estimates. It is done for individual samples. Next, their relationship with the porosity and the formation factor is tested. Furthermore, these values are compared with the estimates from Archie's formula based on electrical properties of the 100% saturated samples[5].

## Connectivity Estimating Techniques - Review

Connectivity is an important aspect of porous rock structure that influences the flow of fluids through rocks during production of oil or natural gas[6]. Connectivity of real rocks has been studied in experiments with pore casts. However, this is a long and destructive type of analysis. Sequential changes can only be tested using different rock samples to represent each instance/step. Furthermore, these different samples have different properties, thus affecting the results and the interpretation.

Other techniques for examining pore connectivity are based on microscopic images of the pores in 2D (thin-sections) of the rock[7]. The thin-sections are analyzed for pore shapes and their size distribution, from which pore connectivity and various properties are inferred. The information obtained this way has little value because 3D properties are estimated from 2D images. New direct techniques include X-ray, CAT scan and NMR imaging[8]. These techniques can be applied to visualize actual rock samples undergoing tests in which fluids are pushed through under pressure, and their output volumes are measured. Such tests do not reveal the structure and connectivity on the pore level. The direct techniques are expensive, provide only for the limited changes in the experimental conditions, and it is difficult to perform such tests in a fashion which leaves the sample ready for further testing[9]. An extensive review of all definitions including other industries can be found in Behzad Ghanbarian et al[10].

In summary, tortuosities are defined in various ways and can differ greatly in value, but in many published works they are used interchangeably. Alternatively, to Archie's form researchers derived several relationships for the electrical tortuosity in the following form[10].

$$\tau = \phi * FF$$

Specific details depend on rock types and other factors. We applied the above formula to test the geometrical tortuosity estimates for samples created in VPL. The geometrical tortuosity is defined as a relative path length between two faces of the sample. Specifically, it is a ratio of the actual path length to the sample length ($L_{path}$ / $L_{sample}$).

## Geometrical Connectivity using Octrees

Computer modelling is an attractive technique that provides for the non-destructive simulation of porous rocks.  The quality of simulated processes is directly dependent on the quality of the algorithms which find the connectivity of the porous network.  In this work, the simulated porous media are represented by a pointer-based octree.  This data structure allows us to model the connectivity of the pores and tracks fluid penetration within the media in 3D.

Prior to and during simulations at every pressure point an active pore network is identified, which is a set of pore cells that have at this instant a continuous communication to both faces. Initially the active porosity is equal to the effective porosity (the whole interconnected porosity of the sample). However, during simulations with fluids the active pore space changes due to the percolation and trapping mechanisms. This is a unique feature of our software.

The identification of the active network starts with any set of pores on one of the model's sides and marking them as connected and visited by a 'walker'. Each new marked octants/cell and all their empty neighbors are marked in the same way in a recursive process. Each of these processes ends when the visited tree node is a part of a pore on the sample opposite side to where process started. These paths get a count number and their length are calculated along the above process by adding the current octant size[7] that is traversed into the accumulated path length. The above process ends when there are no more pore cells left on the starting face that are still 'not visited'.

## Results

20 rock samples already used to develop Archie's 'm' and 'n' exponents[5] had their tortuosity estimated. 13 of samples had homogeneous structures while remaining seven had a correlated pore network with higher pore probability in the horizontal direction. In both cases neighbor searching algorithms gave preference in the horizontal direction when performing any search or

traversal. The effective porosity of samples varied from 2.5% (sample 18) to 30% (sample 9). In addition, we had two samples with extreme parameters (sample 21 and 22 in Table 1 with very low connectivity). In addition, we had resistivity estimates using our in-house developed upscaling process[4]. Applying brine resistivity, we calculated a product of the formation factor and the effective porosity. This product is used as an independent variable in numerical fitting.

Table 1. Porosity (por), resistivity (res), and tortuosity (path_len) data.
Additional abbreviations: **eff**–effective, **max**(imum), **min**(imum), av-arithmetic mean, gav-geometric mean**, har** – harmonic mean, **Correlated** (0/1) for uniform/correlated pore network.

| Index | Correlated | total_por | total_eff_por | r_res_mesh_h | l_nobs | l_max_path_len | l_min_path_len | av_path_len | l_gav_path_len | l_har_path_len |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0.300 | 0.277 | 113 | 13,935 | 878 | 4 | 178 | 109 | 51 |
| 2 | 0 | 0.232 | 0.198 | 309 | 9,553 | 694 | 9 | 232 | 140 | 68 |
| 3 | 0 | 0.232 | 0.198 | 309 | 9,553 | 694 | 9 | 232 | 140 | 68 |
| 4 | 0 | 0.300 | 0.277 | 88 | 13,935 | 878 | 4 | 178 | 109 | 51 |
| 5 | 1 | 0.306 | 0.287 | 36 | 20,189 | 1,341 | 4 | 270 | 141 | 52 |
| 6 | 0 | 0.314 | 0.299 | 38 | 14,004 | 757 | 5 | 198 | 108 | 49 |
| 7 | 1 | 0.319 | 0.304 | 24 | 18,461 | 1,287 | 4 | 330 | 203 | 72 |
| 8 | 0 | 0.318 | 0.304 | 34 | 16,054 | 1,159 | 3 | 204 | 107 | 44 |
| 9 | 1 | 0.323 | 0.306 | 22 | 20,431 | 1,116 | 4 | 289 | 141 | 53 |
| 10 | 0 | 0.312 | 0.299 | 27 | 14,004 | 757 | 5 | 198 | 108 | 49 |
| 11 | 1 | 0.316 | 0.302 | 19 | 17,760 | 1,475 | 5 | 253 | 135 | 53 |
| 12 | 0 | 0.316 | 0.303 | 18 | 14,099 | 2,279 | 5 | 371 | 174 | 54 |
| 13 | 0 | 0.321 | 0.303 | 15 | 17,695 | 1,978 | 6 | 477 | 238 | 75 |
| 14 | 1 | 0.324 | 0.303 | 14 | 19,912 | 2,750 | 6 | 489 | 233 | 69 |
| 15 | 0 | 0.251 | 0.199 | 127 | 12,380 | 1,484 | 7 | 301 | 180 | 80 |
| 16 | 0 | 0.245 | 0.199 | 154 | 2,751 | 508 | 7 | 144 | 83 | 41 |
| 17 | 1 | 0.257 | 0.225 | 39 | 4,683 | 695 | 4 | 159 | 90 | 39 |
| 18 | 0 | 0.202 | 0.025 | 7055 | 243 | 15 | 9 | 12 | 12 | 11 |
| 19 | 0 | 0.211 | 0.113 | 816 | 1,682 | 308 | 9 | 78 | 47 | 31 |
| 20 | 1 | 0.213 | 0.146 | 84 | 2,336 | 383 | 8 | 108 | 69 | 38 |
| 21 | 0 | 0.200 | 0.018 | 21270 | 12 | 13 | 13 | 14 | 13 | 13 |
| 22 | 1 | 0.199 | 0.050 | 156 | 791 | 106 | 14 | 48 | 36 | 29 |

Most tortuosity definitions in published sources are based on a minimum path length or some average value from many observations. Table 1 contains the number of paths, the arithmetic, geometric, and harmonic means of tortuosity followed by the minimum and maximum values for each sample.
These tortuosity estimates defined as the means of the relative path length are much larger than estimates from the Archie's formula that are typically by a single digit (Table 2). On the other hand, the minimum values are in the same range but still greater. Differences between electrical and geometrical tortuosities have been reported before[10].

These bigger numbers represent the geometrical path taken by a walker's path following the shortest but active path in the horizontal direction. In this process the walker's movement is allowed only when a pore is connected to two opposite faces in horizontal direction. Every intermediate step in this search for the path starts in the horizontal direction as it is described in the previous section. A different path search may be implemented if required.

In order to exhaust the current testing process, we created three crossplots and used power formula to fit the tortuosity as a function of the porosity and the formation factor (Figure 1.). Fit statistics indicates that the geometric mean is the best ($R^2$=0.66) but only slightly better than arithmetic mean.

Table 2. Archie's coefficients by the rock type.

| Net type | m | Tortuosity |
|---|---|---|
| All | 2.34 | 2.79 |
| Homogeneous | 2.26 | 4.37 |
| Correlated | 1.92 | 3.05 |

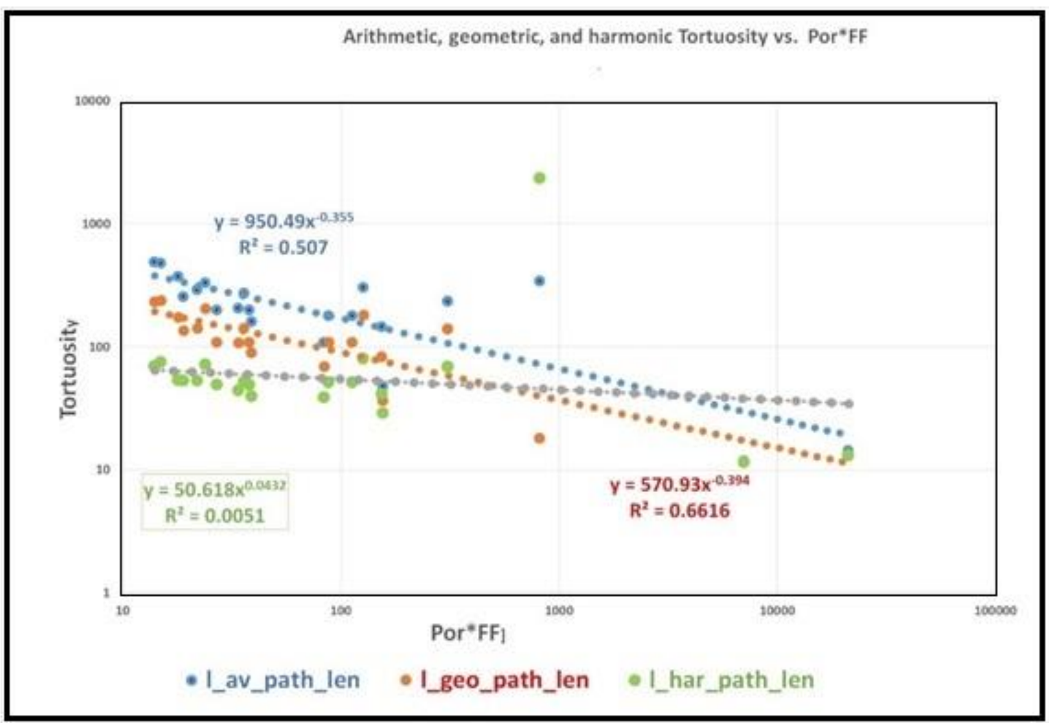


Figure 1. Different geometrical tortuosity means versus the product of porosity and formation factor.

The next two sets of double graphs in Figure 2 and Figure 3 illustrate whether geometrical tortuosity can be used for rock typing. Specifically, Figure 2 shows geometric mean of the tortuosity as a function of the formation factor and the second graph as a function of the product of the formation factor and the sample porosity.

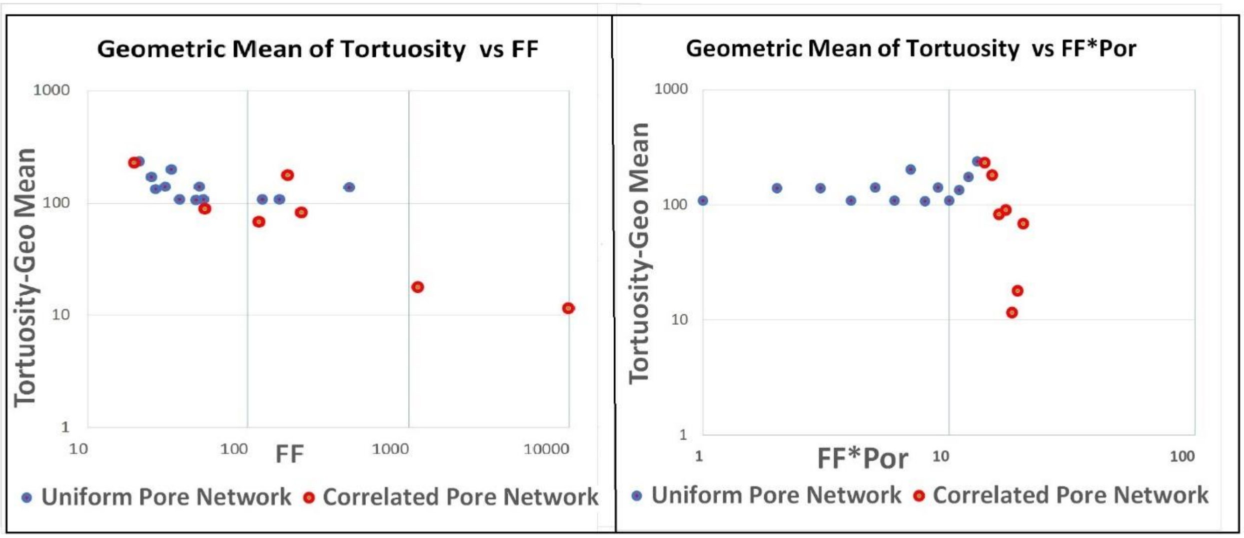


Figure 2. Geometrical mean of tortuosity versus the formation factor and the product of the formation factor and effective porosity.

Both graphs show that a single parameter (porosity, tortuosity, or the formation factor) cannot define the rock type/class. The product of the porosity and the formation factor seems to separate samples with correlated pore network from homogeneous samples along tortuosity axis. Similarly, Figure 3 with the minimum tortuosity for each sample shows the same separation along the FF*ф axis.

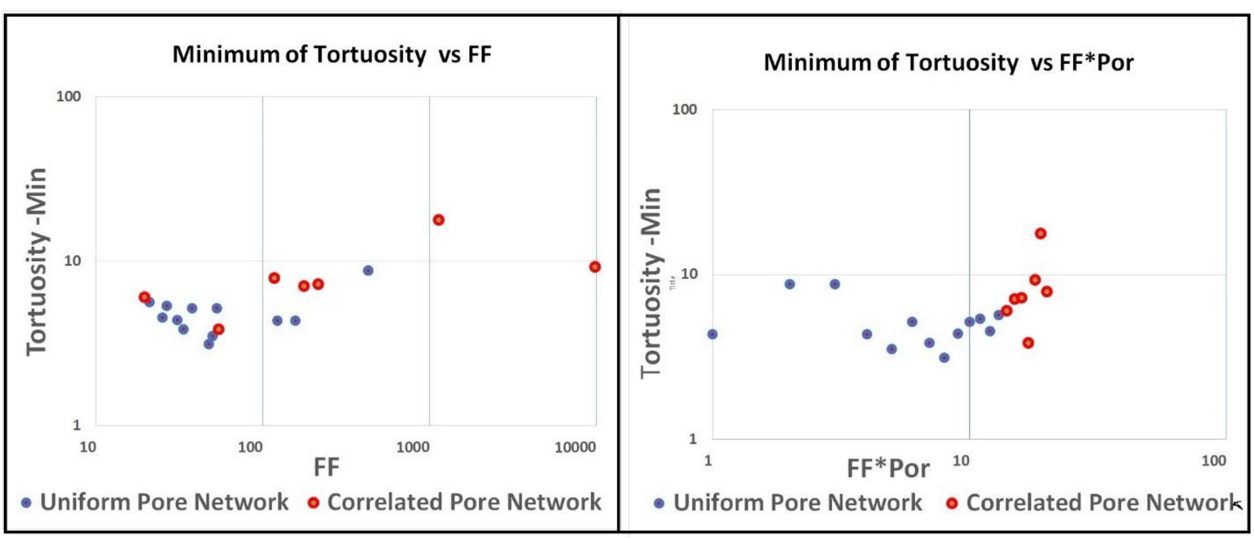


Figure 3. Minimum values of tortuosity versus the formation factor (FF) and the product of the formation factor and the effective porosity (FF*ф).

However, the mean values of the geometrical tortuosity are higher than expected. The minimum values are in a similar range as the electrical tortuosity derived from the Archie's equation[5, 10].

## Novel Information and Key Findings

- Our Virtual Petrophysical Laboratory (VPL) supports building and testing models with porosity from 2% to 30% without observing the percolation threshold.
- We can create and test conductive samples with extremely low porosities in a range from 2% to 3%.
- In VPL we do not use pore equivalent approximations.
- The pore/matrix network is represented and processed using the same element (variable cube size) just having different properties.
- High resolution samples were equivalent to 134 million equal size cells that were compacted into less than 35 million cells of variable size.
- The simulation results show that the geometrical tortuosity cannot fully describe the rock/porous media network type. Thus, the geometrical tortuosity cannot be a substitute for the electrical tortuosity.
- The Archie's electrical tortuosity represents an empirical constant, which changes with data corresponding to the type of porous media and the function type that is implemented in the process.
- The difference between the geometrical and electrical tortuosity may be influenced by the upscaling a single sample versus processing many samples.
- The product of the geometrical porosity and formation factor (resistivity) might be useful for the rock typing characterization.
- A higher resolution (higher number of the octree subdivision levels) is required to simulate micro-porosity, which may improve estimates of the geometrical tortuosity, formation factor, resistivity, and residual saturations observed in the capillary pressure curve simulation[5].

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