Constrained Sparse Spike Seismic Inversion – Then and Now

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Summary

We review the history and concepts underlying constrained sparse spike inversion and show how it is configured with contrasting norms which simultaneously ensure solutions that are optimally sparse while matching the input seismic data. The effects of input low frequency models are considered, comparing traditional and modern methods. The importance of the inversion sparsity term is explored and it is shown that its magnitude contributes significantly to the output bandwidth. Further, interpretations of inversions have evolved with new Bayesian facies analysis techniques. All of these ideas are demonstrated using the Volve data set.

Introduction

AVO inversion and analysis had its roots in the work of Aki and Richards, (1980) which linearized the Zoeppritz equations with a three-term approximation parameterized by P and S velocity and density contrasts accurate to 40 deg. Oldenburg et al. (1983) followed a linear programming approach incorporating an L1 norm on the reflectivities. Shuey's (1985) formulization separated AVO effects into near mid and far angle contributions. Smith and Gidlow (1997) developed a weighted stacking solutions which provided independent estimates of P and S impedance and density. Fatti et al. (1994) re-organized the Aki and Richards equation in terms of P and S impedances and density. Other variations came from Wright (1986), DiSiena et al. (1995), Goodway (1997) and Lancaster and Whitcombe (2000). Meanwhile, Debeye and Van Riel (1990) were experimenting with mixed norm formulations which would lead to the simultaneous inversions of partial-stack data to estimations of P and S impedances and density. This constrained sparse spike approach was described further by Pendrel and Van Riel (1997) and Pendrel et al. (2000). It is this method which is followed in the present work.

Method

Constrained sparse spike inversion algorithms determine elastic properties simultaneously from seismic partial angle or offset stacks toward creating pseudo elastic logs at each seismic trace. The elastic properties usually consist of P impedance, density and a shear measure such as S impedance or Vp/Vs. Other formulations are also possible. Mixed modes are seamlessly handled provided that an accurate alignment has been done. Viable solutions are found by minimizing an objective function consisting of various L1 and L2 norms. Recall that these norms are defined by

$$L_1(x) = \frac{1}{n}\sum_{i=1}^{n} |x_i| \quad \text{and} \quad L_2(x) = \left(\frac{1}{n}\sum_{i=1}^{n} {x_i}^2\right)^{\frac{1}{2}}$$

where X_i is any property which we seek to measure at the i_{th} cell. The 'n's are normalizing constants that facilitate the integration of disparate measures and control the importance of individual terms within the objective function.

The seismic L2 norm ($L2_{Seismic}$) is used to minimize the mean square difference between the input seismic partial stacks and the inversion synthetic computed using a set of partial-stack wavelets. The $L2_{Seismic}$ normalizing constants can conveniently be re-cast in terms of signal-to-noise ratios for each partial stack. Next, an L1 contrast norm is added to minimize the sum of the absolute values of the reflection coefficients ($L1_{RC}$). The $L1_{RC}$ norm promotes a simple solution, both in terms of the magnitude of the reflection coefficients and their numbers. It is immediately apparent that the $L1_{RC}$ and $L2_{Seismic}$ terms act at cross purposes. $L2_{Seismic}$ seeks to match the seismic while $L1_{RC}$ wants a simple solution. Changing the relative importance of these terms can affect the inversion solution in a significant way. Although various algorithms might deal with this contradiction differently, it is convenient to define an $L1_{RC}$ Weight which effectively controls the importance of $L1_{RC}$ vs $L2_{Seismic}$. Its value will be data-dependent and a subject of testing. Its choice has a strong influence on the bandwidth of the final inversion.

There can be several other terms in the objective function, the next most important being a constraint on the low frequencies below the seismic band. These must be provided by the User and have been the subject of much discussion over the years. Simple low frequency models (LFMs) based on well log interpolation and extrapolation are popular and produce the best matches at the well locations. The concern is that they can be inaccurate away from wells. Further, the inclusion of a bad log curve can create havoc in the resulting models. Data-driven LFM techniques have demonstrated improvements. These rely on some sort of seismic attribute or the results of an initial inversion to drive the interpolation between wells. Recently, PDF Transform technology (Pendrel and Schouten, 2020, 2023, Pendrel, 2024) has been used to determine optimum LFMs from inversion or directly from the input seismic. Regardless of the approach, another L1 term ($L1_{LFM}$) is included in the objective function to softly constrain the solutions to follow the LFMs below and about the overlap point with the seismic (merge frequency).

Another issue in obtaining viable solutions arises from the estimation of density. Angles of 50-55 deg. are typically required to achieve desired results. When these data are not available, an additional L1 term ($L1_{G}$) softly constraining the output density to a Gardner relation has been effective. Specific Gardner parameterization can be derived from log data. When the data are either noisy or severely angle-limited, another L1 mudrock term ($L1_{MR}$) can be useful in stabilizing the estimation of the shear measure based on an observed relation between P and S velocities.

Constraining the solutions to be somewhat smooth along local dip can act as a powerful noise reduction agent. Therefore, an L1 constraint ($L1_{Smooth}$) term is optionally added to softly limit the variation in elastic properties from one trace to the next. The magnitudes of the limits are User-controlled. The minimization of the objective function can be handled in a variety of ways, usually iteratively. That is another topic and beyond the scope of this paper.

Interpretations and analyses of seismic inversions have typically been accomplished through Bayesian facies analysis (Pendrel et al., 2006). It is done in a straightforward manner by first creating multi-dimensional probability density functions (PDFs) corresponding to the inversion outputs for each of the proposed elements in the facies set. This can be done using logs, analogues or models. Then, prior facies models can be combined with the PDFs to determine the probabilities of occurrence of each of the members of the facies set at each 3D cell across the project. The approach has been improved over time to include a significant number of improvements including:

- Unique analyses per geologic layer
- Bias and uncertainty estimates and incorporation

- Depth-dependent PDFs – parametric or binned
- 3D priors with prior uncertainties
- Confidence estimates
- AI facies probabilities from Logistical Regression
- Bayesian facies design from petrophysical properties

Results

The procedure is demonstrated using the Volve data set kindly made available by Equinor. The Volve field produced from the Hugin formation structural high from 2008 to 2016 with a cumulative production of 63 million barrels of oil (Weijermars, 2024). Peak production was 56,000 bbl/day. The Hugin is set in a shallow marine environment with transgressive sequences ranging from 5 m to 126 m thick. Estuarian channels are present in the middle Hugin where porosities of 20% and permeabilities of 890 mD are common. GOR ranges from 111 to 157 (Hallam et al., 2020).

The input data consisted of three partial-angle stacks: 3-18 deg., 18-32 deg. and 32-41 deg. The seismic band extends from approximately 10 Hz to 60 Hz. Partial-stack alignment was done and wavelets were estimated for each partial-stack using the available logs. Four logs were available although each required significant corrections, often involving shear sonic modeling.

The first step is to define the relevant facies using Bayesian inference. The facies design has been based on gamma ray and effective porosity. Certainly more and different properties could have been selected. The advantage of the Bayesian approach is that facies probability log curves are created in addition to the facies curves themselves. Finally, the facies are examined through an elastic window corresponding to the inversion outputs. Figure 1 shows the results viewed both from the petrophysical and elastic perspectives. Seven facies were defined:

- High Quality sand
- Low Quality sand
- Silty Sand
- Silt
- Shale 1
- Shale 2
- Shale 3

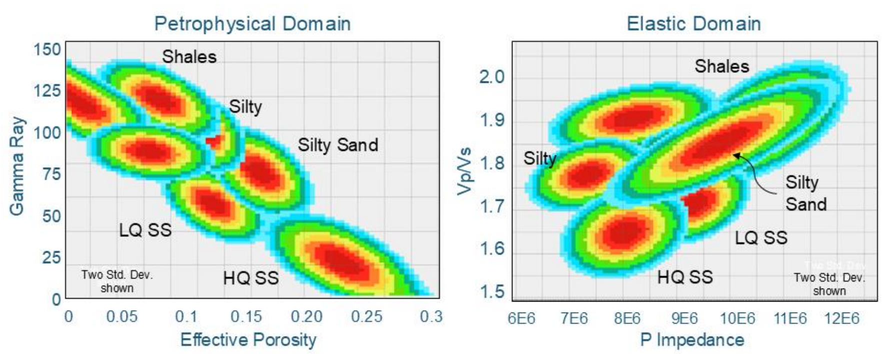

Figure 1: Representations of the facies set in the petrophysical (left) and elastic (right) domains

Note that these facies names are just labels – the real meaning of each is in the petrophysical properties from which they are defined. Importantly, the HQ sand is resolvable in the elastic domain, indicating that inversions should be effective in mapping these facies.

The inversions were completed using all three partial stacks and their corresponding wavelets. The $L1_{RC}$ Weight was set to 0.5, the Gardner constraint was turned on and the mudrock and noise constraints were turn off. The merge frequency was 10 Hz. There were two approaches to creating the LFMs. First, elastic models were constructed from the simple horizon-based interpolation of the corrected logs. Second, constant LFMs were created from an edited average over the logs resulting in a spatially-invariant but structurally-compliant LFM. Bayesian facies analyses including net pay calculations in the Hugin were done from the inversion for both methods. The facies from the constant LFM approach were next used to create new elastic models using the PDF Transform method described by Pendrel (2024). These were used as new sources of LFMs for a repeat of the inversion. The procedure can be continued with successive iterations until the results converge. After each iteration, facies analysis was done and Hugin net pay calculated.

Figure 2 compares Hugin net pay estimations from inversions using the log interpolation LFM with iterations of the PDF Transform method. In all cases, the net pay calculations were probabilistic. Only High Quality sand facies were counted where their probabilities of occurrence were 0.9 or greater. Further, their contributions were weighted by their probabilities of occurrence. Clearly, the PDF Transform method indicates more sand and spatial detail, an observation which increases and then levels of with succeeding iterations. Note that the observed increases do not necessarily reflect additional found sand but rather increases in the confidence of the presence of High Quality sand facies with respect to the other facies types.

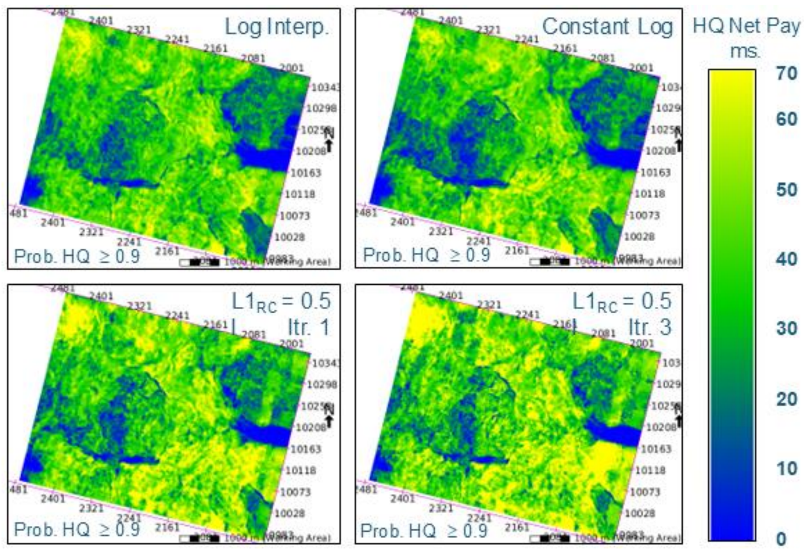

Figure 2: Hugin net pay for various LFMs: Log Interpolation (upper left), Constant Log (upper right), PDF Transforms – iteration #1 (lower left) and iteration #3 (lower right)

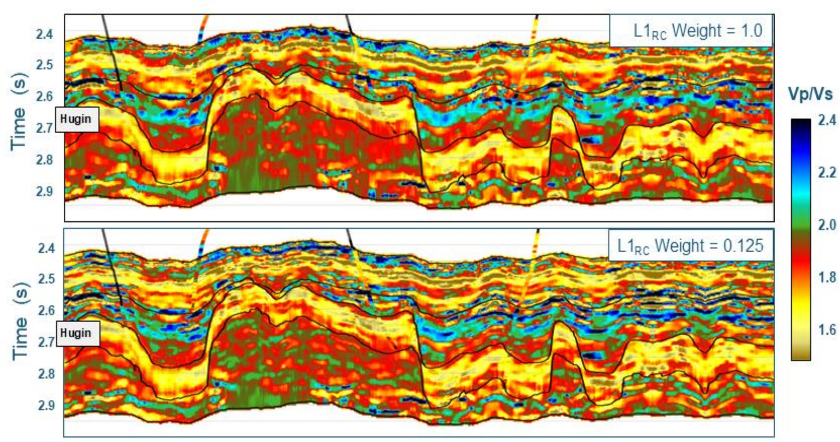

Figure 3: Vp/Vs inversions for values of the $L1_{RC}$ Weight set to 1.0 (upper) and 0.125 (lower). Note the increased detail in the lower panel.

Next, the effects of changing the $L1_{RC}$ Weight were investigated. Decreasing the weight reduces the effect of that term, permitting the reflection coefficients to increase in number and magnitude without affecting the minimization of the objective function. At the same time, more importance is effectively placed on matching the seismic, at the higher and noisier frequencies. The result is more detail in the inversion solution and greater bandwidth. This can be seen in Figure 3 which compares Vp/Vs from inversion for $L1_{RC}$ Weights of 1 and 0.25. Figure 4 shows Vp/Vs relative inversion spectra about the Hugin interval for various values of the $L1_{RC}$ Weight. As the weight is decreased, more flexibility is provided in the design of the inversion reflection coefficients toward the minimization of the $L2_{Seismic}$ term. The algorithm attempts to model more of the higher and noisier frequencies until the noise takes over and the signal is lost. Inversion uncertainty estimates have demonstrated that the prices to be paid for extended bandwidths are increases in uncertainty which must be weighed against the benefits of higher frequencies. This is illustrated in Figure 5 which shows how the relative uncertainty increases as the $L1_{RC}$ Weight is decreased. The uncertainty was estimated by measuring the standard deviations of PDF representing a histogram of residuals to the regression between the inversion pseudo logs and high-cut filtered well logs. The figure shows that Vp/Vs is more affected than P Impedance.

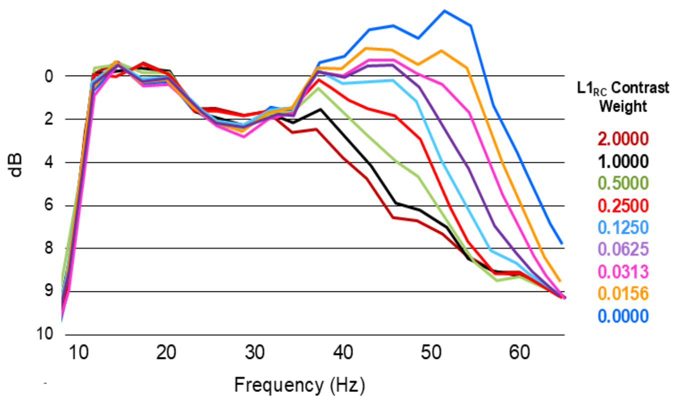

Figure 4: Vp/Vs relative inversion spectra about the Hugin for various values of the $L1_{RC}$ Weight. Smaller values result in a broadening of the inversion spectra.

Conclusions

It has been shown how constrained sparse spike inversion and Bayesian facies analysis can be combined to provide effective analyses of reservoir properties. PDF Transform technology was used to create superior low frequency models resulting in inversions with better and more reliable detail. It was also demonstrated how the relative influences of the $L1_{RC}$ and $L2_{Seismic}$ norms in the inversion objective function could be leveraged to control the output inversion band toward the inclusion of higher frequencies and increased resolution. As the weight of the L1 contrast norm is decreased, the algorithm is able to model the seismic in greater detail at higher and noisier frequencies while continuing to effectively minimize the objective function.

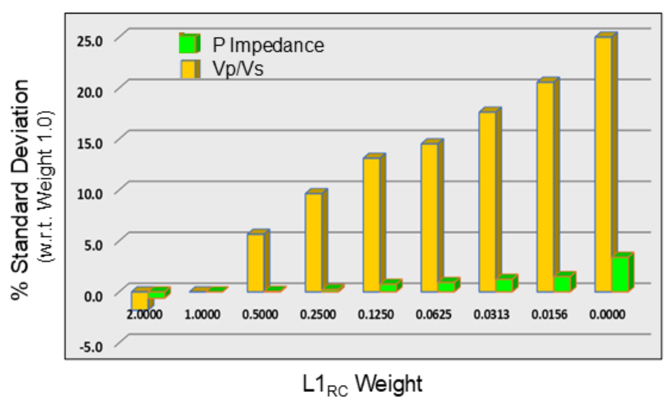

Figure 5: Estimated inversion uncertainty is shown as a function of $L1_{RC}$ Weight. As this contrast weight is decreased, higher and noisier frequencies are modelled in the inversion but at the cost of increased uncertainty. Vp/Vs is more affect than P Impedance.

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References

Aki, K., and Richards, P.G., 2002, Quantitative Seismology, 2nd Edition: W.H. Freeman and Company.

Debeye, H.W.J., van Riel, P. (1990): Lp-norm deconvolution, Geophysical Prospecting 38,381 https://doi.org/10.1111/j.1365-2478.1990.tb01852.x

DiSiena, J.P., Parsons, B.E., Hilterman, F.J. (1995), "Two-term inversion from AVO to detect combined effect of porosity and gas in a cretaceous sand reservoir," SEG Technical Program Expanded Abstracts : 111-114. https://doi.org/10.1190/1.1887313

Fatti, J.L., Smith, G.C., Vail, P.J., Strauss, P.J., Levitt, P.R., (1994), "Detection of gas in sandstone reservoirs using AVO analysis: A 3-D seismic case history using the Geostack technique," Geophysics 59: 1362-1376. https://doi.org/10.1190/1.1443695

Goodway, W. et al., 1997, Improved AVO Fluid Detection and Lithology Discrimination using Lame Parameters; $\lambda\rho$, $\mu\rho$ and λ/μ Fluid Stack, from P and S Inversions, 67th SEG Ann. Mtg. Abs. p183 https://doi.org/10.1190/1.1885795

Hallam T., MacBeth C., Chassagne R, Armini H.,4D seismic study of the Volve Field, First Break, v38, Feb, 2020, p 59-70 10.3997/1365-2397.fb2020011

Lancaster, S., Whitcombe, D., (2000), "Fast-track 'coloured' inversion," SEG Technical Program Expanded Abstracts : 1572-1575. https://doi.org/10.1190/1.1815711

Oldenburg, D.W., Scheuer, T., Levy, S., 1983, Recovery of the acoustic impedance from reflection seismograms, Geophysics, 48, 1318-1337 https://doi.org/10.1190/1.1441513

Pendrel, J., van Riel, P. (1997): Methodology for Seismic Inversion - A Western Canadian Reef Example, CSEG Recorder, 22, #5, p.5

Pendrel, J., H. Debeye, R. Pedersen-Tatalovic, B. Goodway, J. Dufour, M. Bogaards, and R. Stewart, (2000), "Estimation and interpretation of P and S impedance volumes from simultaneous inversion of P-wave offset seismic data," SEG Technical Program Expanded Abstracts : 146-149. https://doi.org/10.1190/1.1815683

Pendrel, J., Mangat, C., Feroci, M., 2006, Using Bayesian inference to compute facies-fluids probabilities: CSEG GeoConvention Abstracts

Pendrel, J., Schouten, H., 2020, Facies – the drivers for modern inversions, The Leading Edge, 39, 102-109 https://doi.org/10.1190/tle39020102.1

Pendrel, J., Schouten, H., 2023, Direct modeling of reservoir properties from seismic partial-angle stacks: SEG/AAPG Int. Mtg. for Applied Geoscience & Energy, SEG-2023-3915405, https://doi.org/10.1190/image2023-3915405.1

Pendrel, J., 2024, Direct modeling of reservoir properties with Feynman path integration: SEG Technical Program Expanded Abstracts : 1826-1829. https://doi.org/10.1190/image2024-4094928.1

Shuey, R.T., 1985, A Simplification of the Zoeppritz Equations, Geophysics 50. p.609-614 https://doi.org/10.1190/1.1441936

Smith, G.C., Gidlow, P.M., 1987, Weighted Stacking for Rock Property Estimation and Detection of Gas, Geophysical Prospecting 35. p.993-1014 https://doi.org/10.1111/j.1365-2478.1987.tb00856.x

Weijermars R., Fast production and water-breakthrough analysis methods demonstrated using Volve Field data, Petroleum Research 2024, v9 #3, p327-346 https://doi.org/10.1016/j.ptlrs.2024.03.001

Wright, J., (1986), Reflection coefficients at pore-fluid contacts as a function of offset, Geophysics v51, p 1858-1860 https://doi.org/10.1190/1.1442232