geoconvention
Calgary • Canada • May 11-13 2026

Enabling curvilinear finite-difference in Devito for elastodynamic simulations in complex topography

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Summary

Devito is an open-source domain-specific language for constructing finite-difference solvers, originally designed for simulations on Cartesian grids. However, conventional Cartesian schemes struggle to represent irregular free-surface topography, where stair-step boundaries introduce numerical artifacts that degrade wavefield accuracy. We extend Devito to support finite-difference modeling in orthogonal curvilinear coordinates (FD-OCC) for elastic wave propagation, using a tensorial representation of the velocity–stress system. This approach preserves the structure and efficiency of staggered-grid methods while allowing the free surface to coincide with an arbitrary topographic profile. The formulation is fully integrated into Devito's symbolic framework and requires only spatially varying metric terms derived from the coordinate mapping. Validation against spectral-element simulations using a 2D slice of the SEAM Foothills Phase II model shows close agreement in wavefield kinematics and amplitudes, confirming the accuracy of the proposed formulation in complex land settings. These results demonstrate that the proposed FD-OCC implementation broadens Devito's applicability to elastic modeling and inversion in realistic land environments with significant topographic variations.

Methodology

Elastic wave modeling in curvilinear coordinate systems has been investigated in previous studies to represent complex geometries in a geometrically consistent manner by expressing both the governing equations and the field variables directly in the curvilinear domain (Komatitsch et al., 1996; Shragge and Konuk, 2020; Konuk and Shragge, 2021). In this study, we propose using finite-difference in orthogonal curvilinear coordinates (FD-OCC) because orthogonality eliminates off-diagonal metric terms in the governing equations, removing coupling between coordinate directions and allowing the elastic wave equations to be discretized efficiently using staggered-grid finite-difference schemes. This formulation preserves the ability to conform the computational grid to complex surface topography while maintaining numerical efficiency comparable to Cartesian implementations and enabling the free-surface boundary condition to be imposed directly on a coordinate surface. The complete modeling procedure is summarized in Figure 1, which illustrates the transformation between the physical and computational domains and the sequence of operations used during simulation.

The proposed methodology uses the velocity model defined in the physical domain (x, z), where elastic parameters such as density $\rho(x, z)$ and Lamé parameters $\lambda(x, z)$ and $\mu(x, z)$ are specified. An orthogonal curvilinear coordinate transformation $(x, z) \rightarrow (\xi, \eta)$ is then constructed using a conformal mapping approach based on complex-variable formulations, following the method proposed by Ryskin and Leal (1983). In this approach, one coordinate direction is aligned with the surface topography, while the orthogonal direction is generated consistently through the mapping, ensuring grid orthogonality throughout the domain. This transformation defines a structured computational mesh characterized by the metric scale factors

$$h_\xi = \sqrt{(\partial_\xi x)^2 + (\partial_\xi z)^2},\ h_\eta = \sqrt{(\partial_\eta x)^2 + (\partial_\eta z)^2}.$$

The physical velocity model is mapped onto the computational domain by transforming the elastic parameters and spatial derivatives according to the curvilinear metrics. In the computational domain, the elastic wave equations can be written in compact form as:

$$\partial_t \mathbf{u}(\xi, \eta, t) = \mathcal{C}(\xi, \eta)\mathbf{u}(\xi, \eta, t) + \mathbf{s}(\xi, \eta, t)$$

where $\mathbf{u} = (v_x, v_z, \sigma_{xx}, \sigma_{zz}, \sigma_{xz})$ denotes the wavefield variables (particle velocities and stresses), $\mathcal{C}$ is the spatial differential operator that includes metric terms arising from the coordinate transformation, and $\mathbf{s}$ represents the source term. These equations are implemented and discretized using Devito (Louboutin et al., 2019), which provides a symbolic framework for expressing and generating finite-difference operators on orthogonal curvilinear grids. Wave propagation is performed entirely in the computational domain, where the structured grid enables efficient and stable time stepping while honoring the topographic boundary. After the simulation, the modeled wavefield or recorded data are transformed back to the physical domain using the inverse coordinate mapping, allowing the results to be analyzed and compared with simulations performed in Cartesian coordinates.

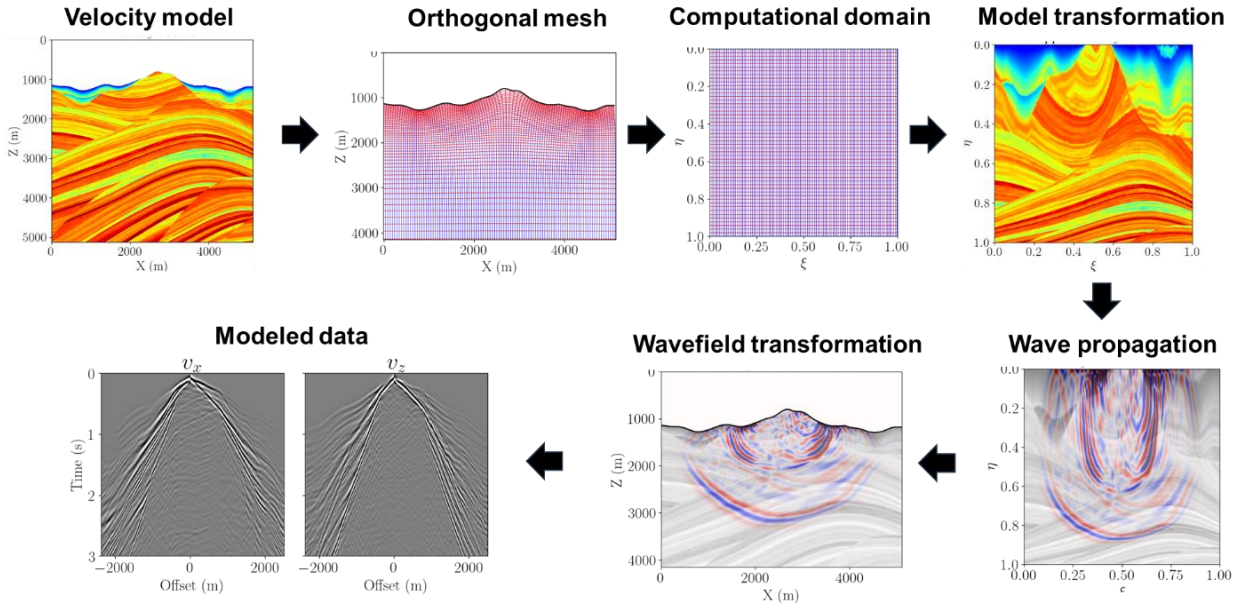


Figure 1. Workflow for elastic wave modeling using finite-difference in orthogonal curvilinear coordinates

Results

The FD-OCC method was evaluated using a 2D slice of the SEAM Foothills Phase II model, which includes strong topographic relief and significant near-surface elastic heterogeneity. The orthogonal curvilinear mesh was generated to conform exactly to the irregular free surface, using a grid of 2001×2001 cells. The elastic parameters V_P, V_S, and density ρ vary spatially throughout the model, introducing complex wave phenomena associated with near-surface scattering and mode conversions. The acquisition geometry consists of 401 receivers deployed on the surface

between 120 m and 5000 m along x-axis, with a spacing of 12.2 m. A 10 Hz Ricker wavelet source was placed at x=2500 m and 10 m depth.

Figure 2 shows a comparison of modeled particle-velocity shot gathers obtained with the FD-OCC implementation and the reference SPECFEM2D (Tromp et al, 2008). The FD-OCC results reproduce the main kinematic features observed in the spectral-element solution, including direct arrivals, strong surface-wave energy propagating along the topographic surface, and complex later arrivals associated with scattering and mode conversions in the heterogeneous near surface. Phase coherence is preserved across the full receiver spread, indicating that the orthogonal curvilinear grid accurately captures wave propagation effects induced by the irregular free surface.

Minor amplitude and waveform differences are observed at the surface waves arrivals, where wavefields interact repeatedly with strong topographic variations and heterogeneous elastic contrasts. These differences are attributed to the distinct numerical discretization employed by the finite-difference and spectral-element methods rather than deficiencies in the curvilinear formulation. Overall, the close agreement between FD-OCC and SPECFEM2D in Figure 2 demonstrates that the proposed approach provides stable and physically consistent elastic wavefield simulations for complex land models such as the SEAM Foothills, while maintaining the efficiency advantages of structured finite-difference grids.

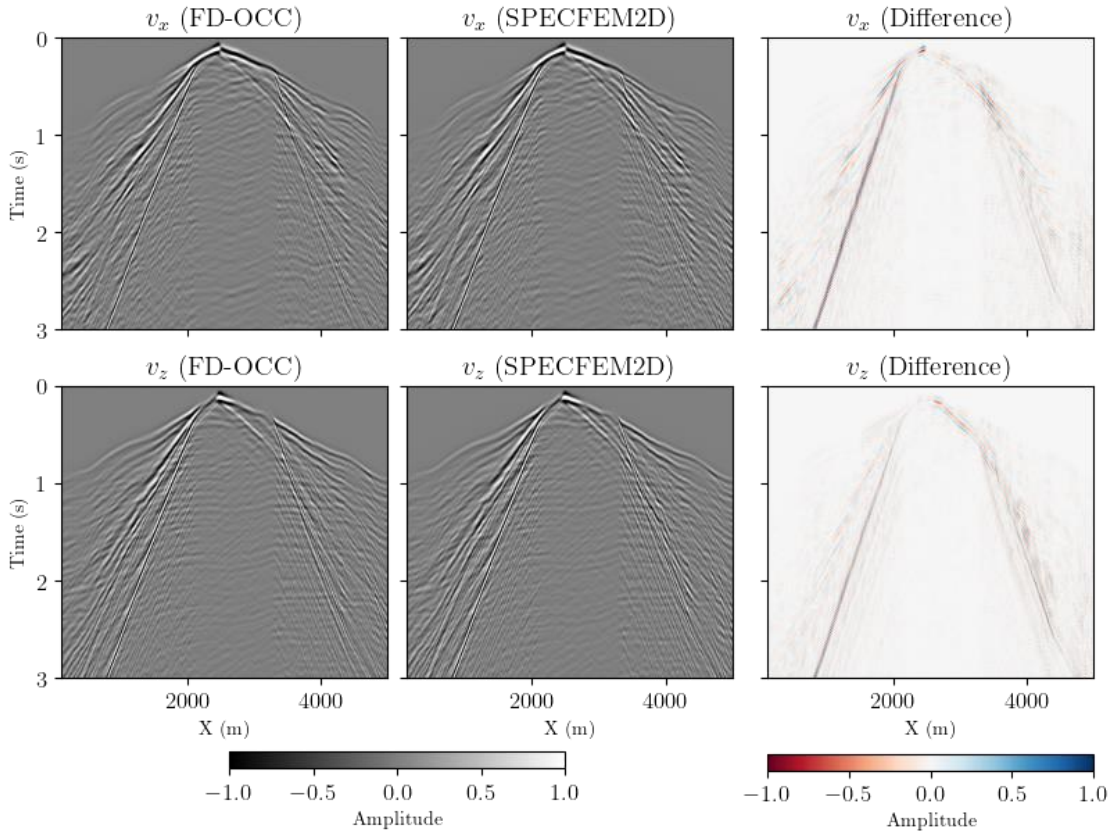


Figure 2. Comparison of modeled particle-velocity shot gathers from FD-OCC and SPECFEM2D simulations. The data have been clipped to 20% of the maximum amplitude for visualization purposes.

Acknowledgements

This work was funded by CREWES industrial sponsors, and NSERC (Natural Science and Engineering Research Council of Canada) through the grants CRDPJ 543578-19 and ALLRP 606155-25. Further support of this work was provided by Emissions Reduction Alberta through the ACT4-SPARSE project.

References

Komatitsch, D., Coutel, F., and Mora, P., 1996, Tensorial formulation of the wave equation for modelling curved interfaces: Geophysical Journal International, 127, No. 1, 156–168.

Konuk, T., and Shragge, J., 2021, Tensorial elastodynamics for anisotropic media: Geophysics, 86, No. 4, T293–T303.

Louboutin, M., Lange, M., Luporini, F., Kukreja, N., Witte, P. A., Herrmann, F. J., Velesko, P., and Gorman, G. J., 2019, Devito (v3. 1.0): an embedded domain-specific language for finite differences and geophysical exploration: Geoscientific Model Development, 12, No. 3, 1165–1187.

Ryskin, G., and Leal, L., 1983, Orthogonal mapping: Journal of Computational Physics, 50, No. 1, 71–100.

Shragge, J., and Konuk, T., 2020, Tensorial elastodynamics for isotropic media: Geophysics, 85, No. 6, T359–T373.

Tromp, Jeroen, Dimitri Komatitsch, and Qinya Liu. 2008. “Spectral-Element and Adjoint Methods in Seismology.” Communications in Computational Physics 3 (1): 1–32.