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Simultaneous Denoising and Statics calculation through Proximal Alternating Minimization

Alejandro Quiaro, Joaquin Acedo and Mauricio D. Sacchi

Signal Analysis and Imaging Group (SAIG), Department of Physics, University of Alberta

Summary

A mathematical formalization of the simultaneous denoising and static calculation problem is presented within the framework of the Proximal Alternating Linearized Minimization. The Proximal Alternating Linearized Minimization is a type of proximal Gauss-Seidel that allows solving the joint inversion of two variables for cost functions with smooth and non-smooth parts by taking alternating proximal minimization steps. By recognizing that statics and random noise are spectrally indistinguishable, a denoiser can be used as an engine for statics calculation. A generalized signal model is discussed, and then a cost function that promotes sparsity and imposes a surface-consistency constraint is minimized. The result is a dataset with maximized lateral predictability and enhanced sparsity, with static corrections purely data-based (no first-break picking).

Introduction

Denoising and static calculations are traditionally treated as separate problems and handled in distinct processing steps (Yilmaz, 2001). While denoising algorithms like FX decon assume a stationary, aligned wavefield (Canales, 1984), residual statics require a pilot trace or reference stack to estimate statics and often struggle in the presence of low SNR (Ronen and Claerbout, 1985). In the simultaneous denoising and static estimation, the signal and the time shifts are linked, and the noise is not seen as just additive random energy, but also coherent misaligned energy; therefore, minimizing noise under this assumption can only be achieved by correctly estimating the static shifts (Traonmilin and Gulunay, 2013; Stanton et al., 2013; Quiaro and Sacchi, 2023). This makes it clear that a denoiser can serve as an engine for static calculations.

In this work, we combine a generalized signal model that accounts for the presence of statics using the sparsity assumption (Gholami, 2013), and a projection onto the surface consistent space (Quiaro and Sacchi, 2023). Both constraints are handled by minimizing the cost function using a Proximal Alternating Linearized Inversion (PALM) algorithm (Bolte et al., 2014). The algorithm is applied on a complex 2D seismic line from the South-Sub Andean.

Statics as noise

The near-surface weathering layer fundamentally affects the processing of seismic land data by introducing significant time delays (statics). These time shifts arise from variations in elevation and weathering velocity, which introduce phase distortions that destroy the lateral continuity of reflection events. The literature has clearly established the connection between statics and noise. Gholami (2013) notices that statics manifest as noise-like features in sparse domains. Stanton et al. (2013) reports that statics effectively increase the rank of seismic data matrices, indistinguishable from random noise. Also, Traonmilin and Gulunay (2013), Quiaro and Sacchi (2023), and Alfaraj et al. (2025) have successfully used that coupling to solve fundamentally the same problem;Traonmilin and Gulunay (2013) proposes a statics-preserving prediction error filter, Quiaro and Sacchi (2023) takes this concept one step further by projecting the statics solution into the surface-consistent space. At the same time, Alfaraj et al. (2025) performed simultaneous statics correction and reconstruction. All of

these approaches recognize that the best filtering (or reconstruction) result comes from data with the best alignment. This realization allows moving from a sequence of heuristic processing steps to a formal inverse problem. Also, notice that "statics-preserving" implies leaving the statics in the data, which means the statics need to be estimated to allow filtering. Therefore, although not mentioned in their corresponding works, the convergence of the algorithms by Traonmilin and Gulunay (2013), and Quiaro and Sacchi (2023) is proof of a successful inversion of the statics.

Proximal Alternating Linearized Minimization (PALM)

Different geophysical problems can be solved by defining a model that combines a convex and a non-convex part. Examples of this include L2-L1 problems such as compressive sensing, sparse signal approximation, and blind deconvolution. There is a well-known suite of solvers for this type of problem, such as ISTA, FISTA, and ADMM. PALM deals with another family of problems which have the form

$$J(x,y) = f(x) + g(y) + H(x,y) \tag{1}$$

No assumption of convexity is made for the objective or the constraints. A standard approach to solve this problem would be Gauss-Seidel (Alternating minimization), which works under the strict convexity assumption, and consists of solving the following iterative problem:

$$\begin{aligned} x^{k+1} &= \text{argmin}_x J(x,y^k) \\ y^{k+1} &= \text{argmin}_y J(x^{k+1},y). \end{aligned} \tag{2}$$

If we remove the strict convexity assumption, by adding a non-smooth L1 norm, for example, the problem can be solved considering the proximal regularization of the Gauss-Seidel scheme:

$$\begin{aligned} x^{k+1} &= \text{argmin}_x J(x,y^k) + \frac{\alpha_k}{2}||x-x^k||_2^2 \\ y^{k+1} &= \text{argmin}_y J(x^{k+1},y) + \frac{\beta_k}{2}||y-y^k||_2^2. \end{aligned} \tag{3}$$

This regularized version is called proximal, because we are looking for solutions in the proximity of the previous coordinate.

When the cost function has a smooth and a non-smooth part, the solution of minimizing the sum of the smooth function h and the non-smooth function σ is equivalent to the proximal regularization of h linearized at a given point x^k;

$$x^{k+1} = \text{argmin}\{\langle x-x^k, \nabla h(x^k)\rangle + \frac{t}{2}||x-x^k||_2^2 + \sigma(x)\} \tag{4}$$

which in proximal notation becomes

$$x^{k+1} = \text{prox}_t^{\sigma}\left(x^k - \frac{1}{t}\nabla h(x^k)\right). \tag{5}$$

Notice that if we use this approach to solve each sub-problem of the proximal regularization of Gauss-Seidel (equation 4) we end up with the PALM iterative scheme:

$$
\begin{aligned}
x^{k+1} &= \text{prox}^{f}_{c_k}\left(x^k - \frac{1}{c_k}\nabla_x H(x^k, y^k)\right). \\
y^{k+1} &= \text{prox}^{g}_{d_k}\left(y^k - \frac{1}{d_k}\nabla_y H(x^{k+1}, y^k)\right).
\end{aligned}
\tag{6}
$$

With that in mind, the cost function we are trying to solve is

$$
J(\mathbf{f},\tau) = \sum_{\omega} \underbrace{\|\mathbf{d}(\omega) - \mathbf{T}(\tau,\omega)\mathbf{D}(\omega)\mathbf{f}(\omega)\|_2^2}_{\text{Data Misfit}} + \lambda \sum_{\omega} \underbrace{\|\Phi_x\{\mathbf{D}(\omega)\mathbf{f}(\omega)\}\|_1}_{\text{Sparsity in FK}} + \mathbf{I}_{\mathscr{C}}(\tau) \tag{7}
$$

where $\mathbf{d}(\omega)$ is the observed data vector at frequency ω, $\mathbf{T}(\tau,\omega)$ is the phase-shift operator, $\mathbf{D}(\omega)$ is the spatial data matrix for f-x prediction, $\mathbf{f}(\omega)$ is the prediction filter, Φ_x is the spatial Fourier transform operator ($x \to k$) and $\mathbf{i}_{\mathscr{C}}(\tau)$ is the indicator function for the surface-consistent subspace $\mathscr{C}$. The data misfit term corresponds to the function H in equation 1, while the non-smooth terms are the projection onto the surface-consistent space and the sparsity promotion. In simple words, minimizing this cost function will allow for the inversion of the statics solution that has maximum lateral continuity (in FX) and can be explained with fewer coefficients in a sparsifying transform. The projection onto the surface-consistent space serves as a regularizer, discarding non-physical solutions.

Results

The workflow was applied to a 2D seismic section from the South-Sub Andean. The dataset is located in an area with a complex near-surface, leading to highly variable first-arrival behaviour. A single-trace preconditioning (spectral whitening and AGC) was applied to attenuate surface waves and enhance the energy of the first arrivals.

The elevation statics solution is used as an initial model to kickstart the inverse problem. Four different surface-consistent decompositions were tested: Source-receiver, Source-receiver-offset, Source-receiver-CDP, and Source-receiver-offset-CDP. It was found that Source-receiver-CDP produced the best results. The most crucial hyperparameter that required extensive testing was the sparsity block's regularization term. Relaxed thresholding allows more complexity in the data, whereas harsher thresholding limits the number of admissible dips; therefore, this parameter will be data-dependent, since the level of sparsity required to constrain the statistics depends on how complex the data is. The corrected test shot gathers are shown in Figure 2. Notice the superior first-arrival alignment and the enhanced continuity of the reflection hyperbolas. After comparison with the data corrected by tomographic refraction statics, it was found that the quality of both results was comparable. More importantly, applying the statics correction using the proposed minimization problem took 15 minutes on a single CPU for a 2D seismic line with 383 shots, without requiring any arrival picking.

Conclusions

This work represents a further refinement of the simultaneous denoising and statics calculation problem, adding the sparsity constraint and solving it within a single minimization framework, avoiding heuristic steps. We showed that reasonable static solutions, comparable to refraction statics, can be achieved in a surface-consistent manner without requiring first-arrival picking by using a denoiser as an engine for statics calculations. This algorithm allows for obtaining a good statistics solution, enabling fast-track images in complex areas and bypassing the time-consuming early stages of first-arrival picking. Finally, although not currently implemented, the workflow is embarrassingly parallelizable and therefore has enormous potential to be more computationally efficient.

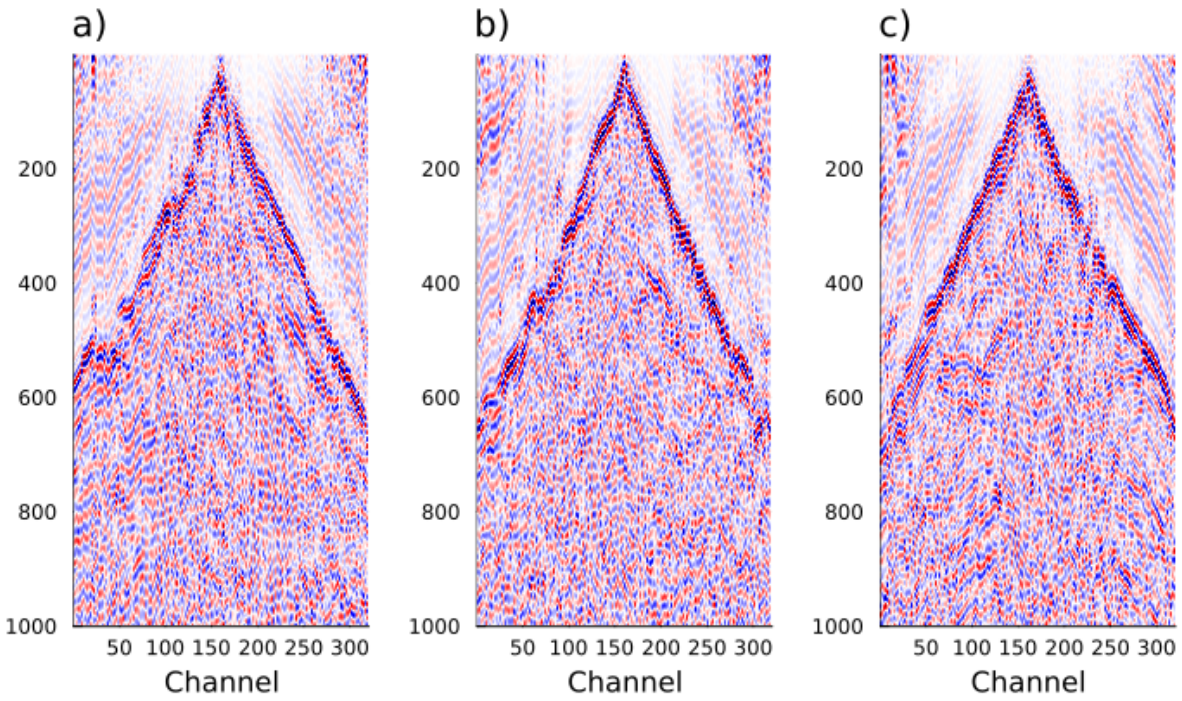


Figure 1: Three different test shots. Notice the complexity of the first arrivals and the lack of continuity in the reflections. Shots have a single trace preconditioning applied to attenuate surface waves and enhance the first arrivals.

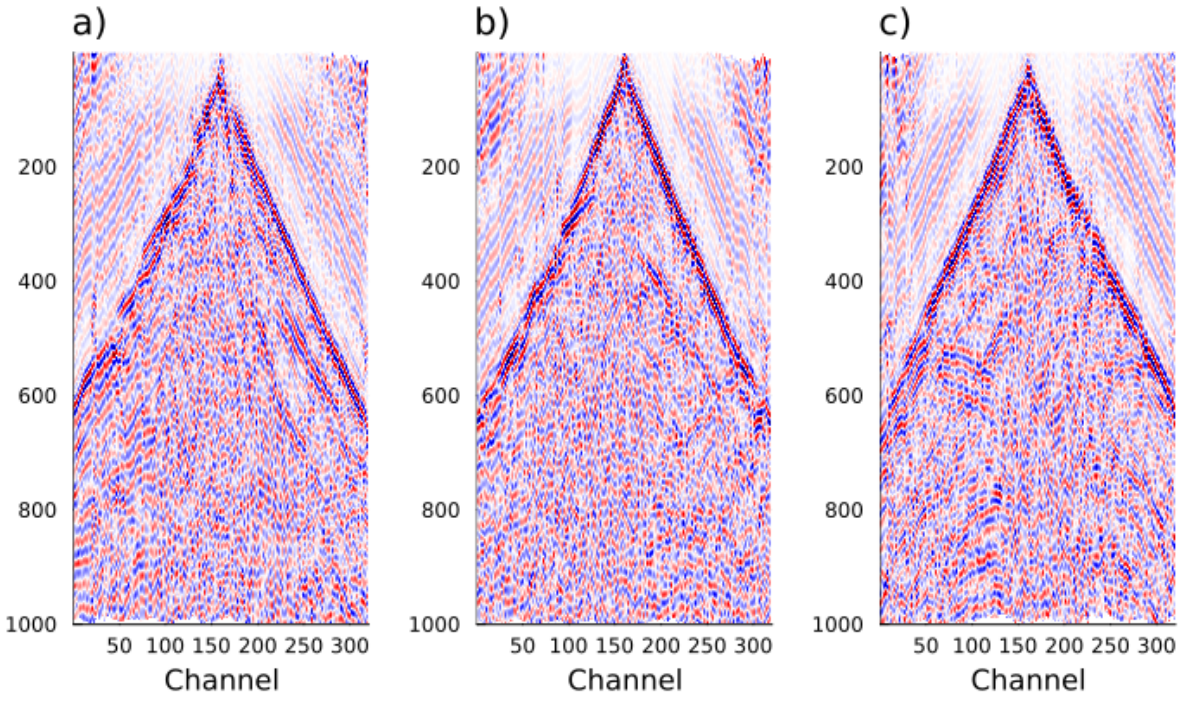


Figure 2: Three different test shots after statics correction. Notice the linearity in the first arrivals and the enhanced continuity of the reflection hyperbolas.

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