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Enhanced FWI using source signature preconditioning

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Summary

Much effort in Full Waveform Inversion (FWI) has focused on designing new objective functions or increasing forward-modeling complexity. In this work, we investigate an alternative strategy based on signal preconditioning through Source Manipulation, applied independently of the inversion. The observed data are preconditioned using a frequency-domain matching filter that suppresses wavelet side-lobe energy. We study both known- and unknown-source scenarios, the latter using either spectral division or by Log-spectral averaging for wavelet estimation.

For a severely inaccurate initial model, the known-source experiment shows that Source Manipulation reduces the misfit by about 50% within the same number of iterations as conventional FWI. In the unknown-source case, source-manipulated data yield higher misfits when using an L2 objective due to wavelet errors; however, this behavior reverses when a zero-lag cross-correlation functional is employed. Among the wavelet estimation methods tested, the Log-spectral averaging provides the most stable results and the best interaction with Source Manipulation.

We show that combining Source Manipulation with the Log-spectral wavelet estimation and a zero-lag cross-correlation objective function significantly reduces the number of iterations and frequency bands required to mitigate cycle skipping in low-frequency inversion, with negligible additional computational cost and without resorting to specialized cycle-skipping-resistant functionals.

Introduction

Full Waveform Inversion (FWI) is a powerful and widely used technique, yet it remains vulnerable to cycle skipping, wavelet uncertainty, and sensitivity to noise and amplitude errors (Virieux and Operto, 2009). Although numerous robust misfit functions have been proposed to address these challenges (Warner and Guasch, 2016; Wu et al., 2014), comparatively little attention has been given to simple time-domain preconditioning strategies that can improve convergence without modifying standard FWI implementations. This study addresses cycle skipping by reshaping the effective source wavelet. Because conventional seismic wavelets exhibit multiple lobes, conventional L2 objectives often lead to highly nonconvex misfit landscapes and unstable inversion. Rather than altering the objective function, we apply a workflow that estimates the source wavelet and reshapes it into a smooth, Gaussian-like form with improved convexity following the workflow proposed by Bao et al. (2023).

The effective wavelet is first estimated using homomorphic (cepstral) deconvolution (Ulrych, 1971; Herrera and Van der Baan, 2012), after which a matching filter is constructed to suppress sidelobes and impose a Gaussian-like signature. Owing to source linearity, this filter is applied directly to the observed data, and FWI is carried out using the modified wavelet. Numerical experiments show that this wavelet-shaping preconditioning improves low-frequency convergence, reduces imaging artifacts, and substantially increases robustness to cycle skipping, allowing FWI to converge from poor initial models with fewer iterations and no changes to existing inversion algorithms.

Theory

Full Waveform Inversion (FWI) aims to recover the subsurface model that explains the recorded seismic data (Tarantola, 1984). Its theoretical foundations are well established in the literature (see Virieux and Operto (2009) for a comprehensive overview). In its classical form, FWI is posed as the constrained optimization problem

$$J(\mathbf{m}) = \frac{1}{2}\sum_{r=1}^{Ng}\sum_{s=1}^{Ns}\int_0^{t_{max}} |\mathbf{p}_{cal}(\mathbf{x_r},t;\mathbf{x_s},\mathbf{m}) - \mathbf{p}_{obs}(\mathbf{x_r},t;\mathbf{x_s})|_2^2 dt \quad \text{subject to } \mathbf{L}(\mathbf{m})\mathbf{p}(\mathbf{x},t) = \mathbf{s}(\mathbf{x}_s,t) \qquad (1)$$

where $\mathbf{m}$ denotes the set of model parameters (e.g., velocity, density, attenuation), $\mathbf{p}_{cal}$ and $\mathbf{p}_{obs}$ are the modeled and observed pressure fields, respectively, $\mathbf{s}$ represents the source term, and $\mathbf{L}$ is the wave-equation operator that incorporates the relevant physics through $\mathbf{m}$.

The source signature $\mathbf{s}$ plays a critical role in shaping the misfit landscape (Bao et al., 2023). Cycle skipping occurs when the predicted data differ from the observations by more than approximately half a wavelength, preventing convergence toward the global minimum (Warner and Guasch, 2016). In exploratory scenarios, initial velocity models are typically too inaccurate to satisfy this condition. As a result, standard FWI workflows rely on multiscale strategies (Bunks et al., 1995) or on cycle-skipping-resistant objective functions, such as envelope-based misfits (Wu et al., 2014), to improve convergence robustness.

To improve the convexity of the misfit function, Bao et al. (2023) propose manipulating the source signature by designing an inverse filter that suppresses wavelet sidelobes, thereby producing a quasi-convex misfit landscape. Consider first the case of a known seismic wavelet, $\mathbf{s}_{obs}$. The observed data $\mathbf{p}_{obs}$ can be written as the convolution of the source signature with the Green's function of the subsurface. Likewise, we define $\mathbf{p}_{des}$ as the data that would be generated by a desired Gaussian-like source wavelet $\mathbf{s}_{des}$:

$$\mathbf{p}_{obs} = \int_0^{t_{max}} \mathbf{G}(\mathbf{x},\mathbf{x}_s;t-\tau)\mathbf{s}_{obs}(\mathbf{x}_s,\tau)d\tau = \mathbf{G} * \mathbf{s}_{obs}, \qquad (2)$$

and equivalently $\mathbf{p}_{des} = \mathbf{G} * \mathbf{s}_{des}$. The source-manipulated data $\mathbf{p}_{des}$ can then be obtained by solving the following optimization problem:

$$J(\mathbf{p}_{des}) = \frac{1}{2}|\mathbf{p}_{obs}\mathbf{s}_{des} - \mathbf{p}_{des}\mathbf{s}_{obs}| + \mathbf{R}(\mathbf{d}_{des}), \qquad (3)$$

where $\mathbf{R}$ denotes a regularization term. This problem admits a closed-form solution via spectral division and can be solved in a single step.

When the source wavelet is unknown, the problem becomes one of blind deconvolution. While several blind approaches exist (Kazemi and Sacchi, 2014; Van der Baan and Pham, 2008; Gholami and Sacchi, 2013), greater stability is achieved by decoupling the workflow into two stages: first estimating the source wavelet, and then performing source manipulation by solving equation 3.

As an alternative to conventional source inversion (Pratt, 1999), we adopt homomorphic wavelet estimation. Homomorphic systems are nonlinear systems that preserve a generalized superposition principle and were first introduced to seismic processing by Ulrych (1971). Consider a seismic trace $\mathbf{p}_i(t)$ generated by the source signature $\mathbf{s}_{obs}(t)$. Mapping the trace to the cepstral domain is achieved through

$$\hat{\mathbf{p}}_i(t) = \mathscr{F}^{-1}(\ln(\mathscr{F}(\mathbf{p}_i(t)))) \tag{4}$$

where $\mathscr{F}$ and $\mathscr{F}^{-1}$ denote the forward and inverse Fourier transforms. This transformation converts time-domain convolution into addition. Assuming white reflectivity, we apply the short-time homomorphic wavelet estimation (STHWE) method of Herrera and Van der Baan (2012), which estimates the source wavelet through log-spectral averaging.

Defining the log-spectral operator $\mathscr{L}$ as

$$\mathscr{L}(\mathbf{p}_i) = \ln(\mathscr{F}(\mathbf{p}_i)) = \hat{\mathbf{P}}_i, \tag{5}$$

the estimated wavelet in the log-spectral domain is obtained by averaging over traces and time windows:

$$\hat{\mathbf{S}}_{obs} = \frac{1}{NM} \sum i = 1^N \sum_{k=1}^{M} \hat{\mathbf{P}}_{ik} = \hat{\mathbf{S}} + \sum i = 1^N \sum_{k=1}^{M} \hat{\mathbf{R}}_{ik}, \tag{6}$$

where i and k index traces and time windows, respectively, and N and M denote their total counts. The estimated source wavelet in the time domain is then recovered via the inverse homomorphic operator

$$\tilde{\mathbf{s}}_{obs} = \mathscr{L}^{-1}(\hat{\mathbf{S}}_{obs}) = \mathscr{F}^{-1}(\exp(\hat{\mathbf{S}}_{obs})). \tag{7}$$

Results

All experiments were performed on a 2D profile extracted from the SEG/EAGE Overthrust model. The velocity model consists of 187×801 grid points with a spatial sampling of $20,\mathrm{m} \times 10,\mathrm{m}$. The data were sampled at 1 ms for a total record length of 3 s (3000 samples). Figure 1 shows selected shots after source manipulation, highlighting enhanced low-frequency content and effective suppression of sidelobe-related artifacts, while preserving the polarity of the primary seismic events.

The observed data were inverted using a multiscale FWI workflow. Figures 2 and 3 compare two inversion strategies applied to the low-frequency band. Figure 2 shows the result of a conventional workflow, where the wavelet is estimated through source inversion in the first iteration. While the main structures and shallow low-velocity zones are partially recovered, strong cycle skipping persists near the model boundaries. In contrast, Figure 3 presents the result obtained using data preconditioned with STHWE followed by source manipulation, yielding a more accurate low-frequency model with sharper velocity contrasts, improved resolution in structurally complex regions, and corrected cycle skipping.

The improvement arises from modifying the misfit landscape: replacing the original wavelet with a Gaussian-like signature suppresses local minima caused by correlations between the main lobe and sidelobes (Bao et al., 2023). As a result, source manipulation enlarges the radius of convergence of FWI, enabling convergence from poorer initial models. Importantly, this preconditioning is applied independently of the inversion loop and requires no modification to existing FWI implementations. Both STHWE and source manipulation incur negligible computational cost, relying only on FFTs and spectral division.

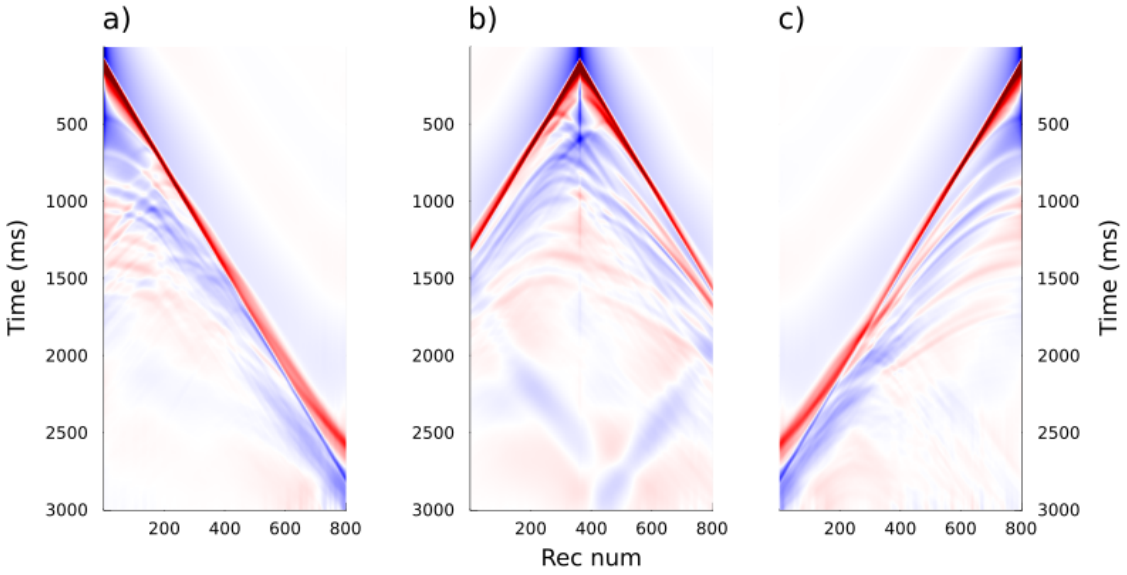


Figure 1: Selected shots of the synthetic observed data, after the preconditioning.

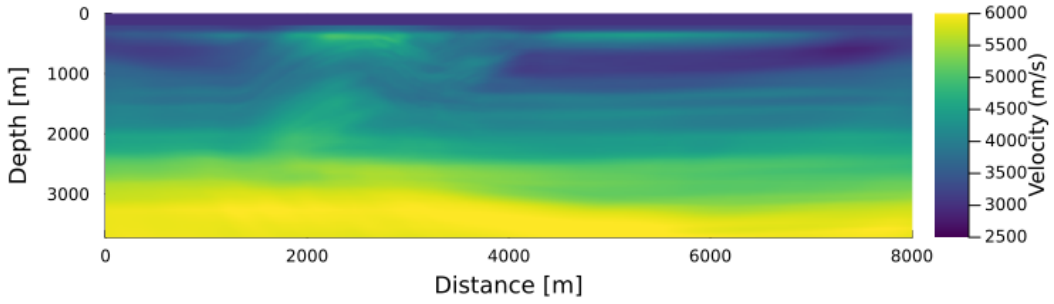


Figure 2: Velocity field obtained by inverting the 1-10Hz frequency band using a conventional FWI workflow.

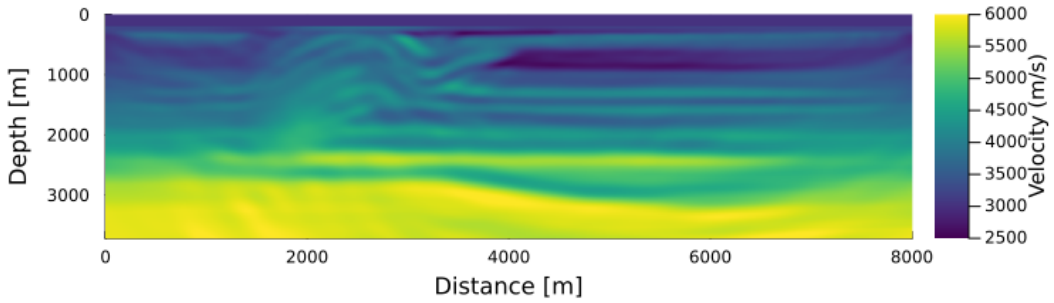


Figure 3: Velocity field obtained by inverting the 1-10Hz frequency band using the proposed preconditioning workflow (STHWE and source manipulation).

Conclusions

We evaluated a preconditioning strategy to reduce cycle skipping in FWI that integrates homomorphic wavelet estimation with source manipulation through matching filters. The proposed workflow limits reliance on aggressive multiscale schemes or specialized objective functions while remaining fully compatible with standard FWI frameworks. Among the approaches considered, STHWE yields the most reliable wavelet estimates for source manipulation, allowing the construction of a stable, Gaussian-like effective source. The method improves phase matching and accelerates convergence even from inaccurate initial models. All additional steps are implemented as low-cost preconditioning, requiring no modification to the inversion algorithm and incurring negligible computational overhead. Future work will focus on assessing robustness in noisy environments and applications to field data.

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