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Fast Iterative Reweighted Least-Squares Method for Robust Reduced-Rank Filtering of Erratic Seismic Noise

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Summary

This paper presents a stable inverse-problem algorithm for robust low-rank seismic denoising via Singular Spectrum Analysis (SSA), designed to handle erratic, non-Gaussian noise. The method builds on the robust bisquare-penalty formulation introduced by Chen and Sacchi (2015) for rank reduction and employs an iterative reweighted least squares (IRLS) framework accelerated by a preconditioned conjugate gradient solver.

The approach retains the linear convergence properties of classical IRLS while achieving greater computational efficiency through a tailored preconditioner. The result is numerically reliable, and the algorithm has effectively downweighed outliers from non-Gaussian noise and mitigates issues such as matrix singularities and null solutions.

The IRLS-PCG algorithm is fast and well-suited for solving large-scale inverse-problems and can be extended to sparsity-promoting regularization. Validation on synthetic and field seismic data—including a dataset from the Western Canadian Sedimentary Basin—demonstrates the method's effectiveness in attenuating erratic noise while preserving signal integrity, outperforming conventional SSA and other related denoising techniques.

Robust reduced-Rank filtering of Trajectory matrix

Chen and Sacchi (2015) adopted Tukey's bisquare loss function to achieve robustness against erratic noise. The robust low-rank factorization of Hankel matrix $\mathbf{M} \approx \mathbf{U}\mathbf{V}^H$ can be represented as:

$$\min_{\mathbf{U}\in\mathbb{C}^{m\times K},\mathbf{V}\in\mathbb{C}^{n\times K}} E(\mathbf{U},\mathbf{V}) = \sum_{i=1}^{m}\sum_{j=1}^{n} \rho\left(\frac{r_{ij}(\mathbf{U},\mathbf{V})}{\sigma}\right) \qquad (1)$$

where K is the target rank and K≪min(m,n), ρ is the Tukey bisquare function, $r_{ij} = m_{ij} - [\mathbf{U}\mathbf{V}^H]_{ij}$, and $\sigma > 0$ is a scale parameter.

Minimization of cost function (1) with regards to $\mathbf{U}^*$and $\mathbf{V}^*$, leads to the following alternate weighted Least-squares equations:

$$(\mathbf{V}^H\mathbf{W}_i\mathbf{V})\mathbf{u}_i = \mathbf{V}^H\mathbf{W}_i\mathbf{m}_i \quad (2)$$

$$\left(\mathbf{U}^H\mathbf{W}_j\mathbf{U}\right)\mathbf{v}_j = \mathbf{U}^H\mathbf{W}_j\mathbf{m}_j \quad (3)$$

where the wight function $w(x) = \frac{\rho'(x)}{x} = \begin{cases} \left[1 - \left(\frac{|x|}{\alpha}\right)^2\right]^2, & |x| \le \alpha \\ 0, & |x| > \alpha \end{cases}$ and α is a tuning parameter. Chen and Sacchi (2015) advocate to use QR factorization method to alternately update **U** and **V** in the IRLS procedure. While QR factorization is effective for synthetic data, some studies suggest it can be unstable with certain field data due to matrix singularities.

IRLS-PCG

To improve algorithm stability, we solved the alternate reweighted Least-square equations (2) and (3) using an accelerated preconditioned conjugate-gradient method (Golub & Van Loan, 2013). The IRLS-PCG preconditioners for these equations are defined below:

$$\mathbf{P}_U = a\boldsymbol{I} + \lambda(\boldsymbol{V}^H\boldsymbol{W}_i\boldsymbol{V}) \quad (4)$$

$$\mathbf{P}_V = a\boldsymbol{I} + \lambda\left(\mathbf{U}^H\mathbf{W}_j\mathbf{U}\right) \quad (5)$$

where a is the mean-diagonal of $\mathbf{V}^H\mathbf{V}$ in equation (4) or mean-diagonal of $\mathbf{U}^H\mathbf{U}$ in equation (5), $\boldsymbol{I}$ is an identity matrix and λ is a regularization parameter.

Algorithm 1 summarizes all steps needed for IRLS-PCG for robust rank reduction filter using Tukey's bi-square penalty function (1). Application to different field dataset have demonstrated that the proposed algorithm is fast, stable and outperformed QR method.

Algorithm 1: IRLS-PCG method for Robust Reduced-Rank Factorization

Input: Hankel matrix $\mathbf{M} \in \mathbb{C}^{m\times n}$, rank K, parameters α,ϵ_{tol},$k_{\max}$

1. **Initialize**: $\mathbf{U}^0, \mathbf{V}^0$(via TSVD of $\mathbf{M}$ or as random), $\sigma^0 = 3.33 \times median(\mathbf{M} - \mathbf{U}^0(\mathbf{V}^0)^H)$
2. **For** $k = 1,2,\dots,k_{\max}$
    a. **Residual computation**: $\mathbf{R}^k = \mathbf{M} - \mathbf{U}^k(\mathbf{V}^k)^H$

b. **Scale update**: $\sigma^{k+1} = 3.33\ median(\text{vec}(\mathbf{R}^k))$, $x_{i,j}^k = \frac{|r_{i,j}^k|}{\sigma^{k+1}}$

c. **Weight computation**: For each(i, j): $w_{ij}^k = \begin{cases} \left[1 - \left(\frac{x_{i,j}^k}{\alpha}\right)^2\right]^2, & |x_{i,j}^k| \le \alpha \\ \varepsilon, & |x_{i,j}^k| > \alpha \end{cases}$

where ε is an exceedingly small number (e.g.,1e-8) introduced to replace zero, and to stabilize the weight matrix w_{ij}^k. This theoretical justification ensures that the weighted Least-squares problem remains well-conditioned while still assigning near-zero weight to outliers — effectively rejecting them.

3. **U-update: For $i = 1, \dots, m$:**
   a. Form $\mathbf{W}_i = \text{diag}(w_{i1}^k, \dots, w_{in}^k)$
   b. Solve $(\mathbf{V}^k)^H \mathbf{W}_i \mathbf{V}^k \mathbf{u}_i^{k+1} = (\mathbf{V}^k)^H \mathbf{W}_i \mathbf{m}_i^T$ via PCG with a conditioner $\mathbf{P}_U$

4. **V-update: For $j = 1, \dots, n$:**
   a. Form $\mathbf{W}_j = \text{diag}(w_{1j}^k, \dots, w_{mj}^k)$
   b. Solve $(\mathbf{U}^{k+1})^H \mathbf{W}_j \mathbf{U}^{k+1} \mathbf{v}_j^{k+1} = (\mathbf{U}^{k+1})^H \mathbf{W}_j \mathbf{m}_j$ via PCG with a conditioner $\mathbf{P}_V$
5. **Convergence check**: If $\| \mathbf{U}^{k+1} - \mathbf{U}^k \|_F < \epsilon_{\text{tol}}$ and $\| \mathbf{V}^{k+1} - \mathbf{V}^k \|_F < \epsilon_{\text{tol}}$, break
6. **Return**: $\mathbf{U}^{k+1}, \mathbf{V}^{k+1}$

Results

Figure (1a) shows a 2D synthetic seismic data that are contaminated by gaussian noise as well as by isolated erratic noise traces (non-gaussian noise). The Fast IRLS-PCG method given in algorithm 1 is applied and effectively attenuates both Gaussian and non-Gaussian erratic noise (figure 1c), producing a significantly cleaned seismic section (figure 1b).

In Figure (2a), the same algorithm is applied to a field dataset from the Western Canadian Sedimentary Basin (WCSB). The input stacked section is heavily contaminated by prominent erratic traces. The final clean stack section, presented in figure (2b), confirms the method's effectiveness in suppressing these erratic events along with underlying Gaussian noise (figure 2c).

Conclusion

The Fast IRLS-PCG method offers a stable solution for attenuating erratic and gaussian noise in seismic data. By using a preconditioned conjugate gradient solver, it avoids the numerical instability encountered with QR-based methods and converges efficiently. Results from application algorithm to field dataset confirm the method's effectiveness in preserving coherent reflection events while removing challenging noise, making it a reliable tool for improving data quality in processing flows.

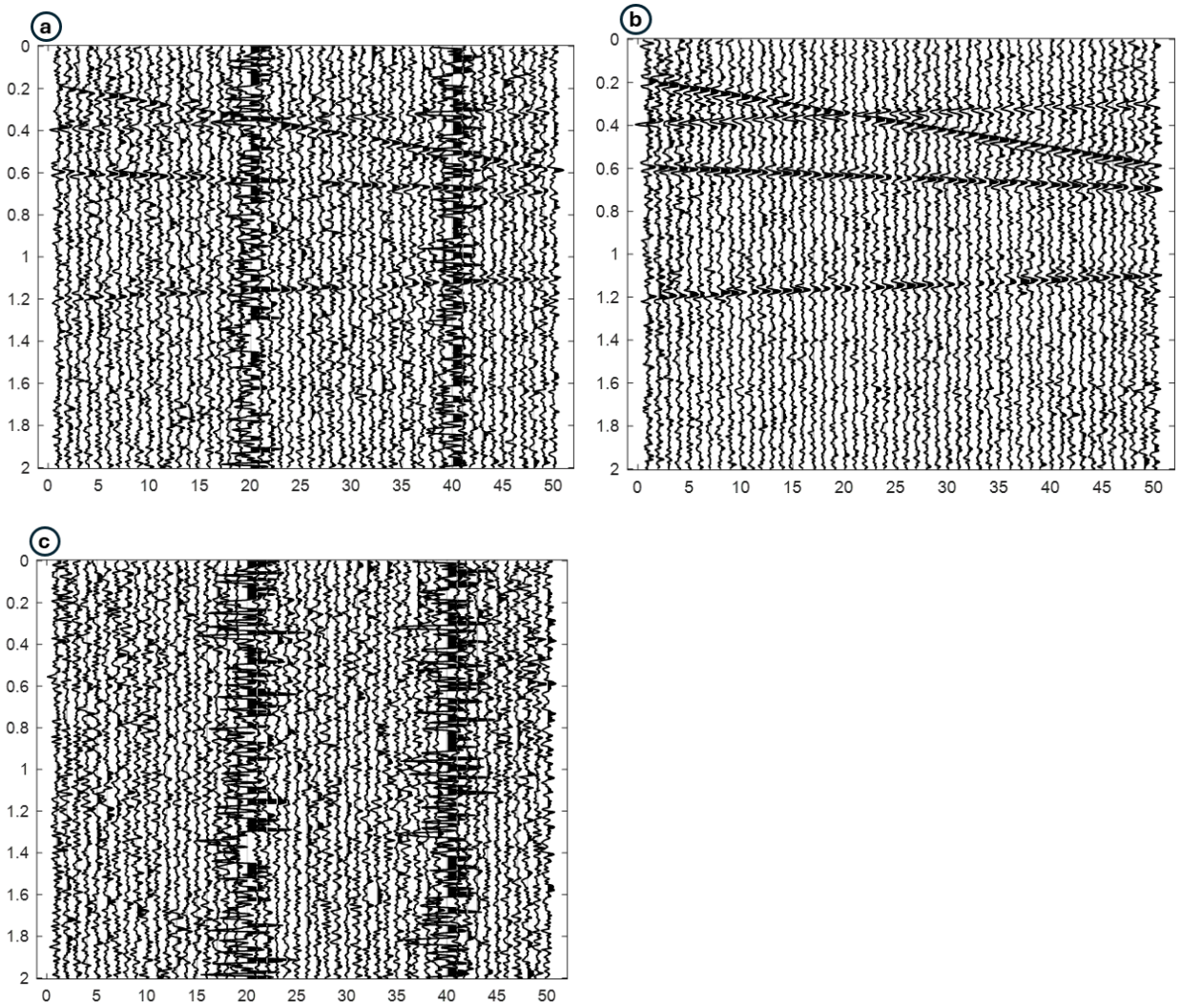


Figure 1: Panel (a) shows the input synthetic seismic data contaminated with both Gaussian and non-Gaussian noise. Panel (b) presents the filtered seismic section obtained by applying the proposed algorithm. Panel (c) depicts the corresponding noise removed during the filtering process.

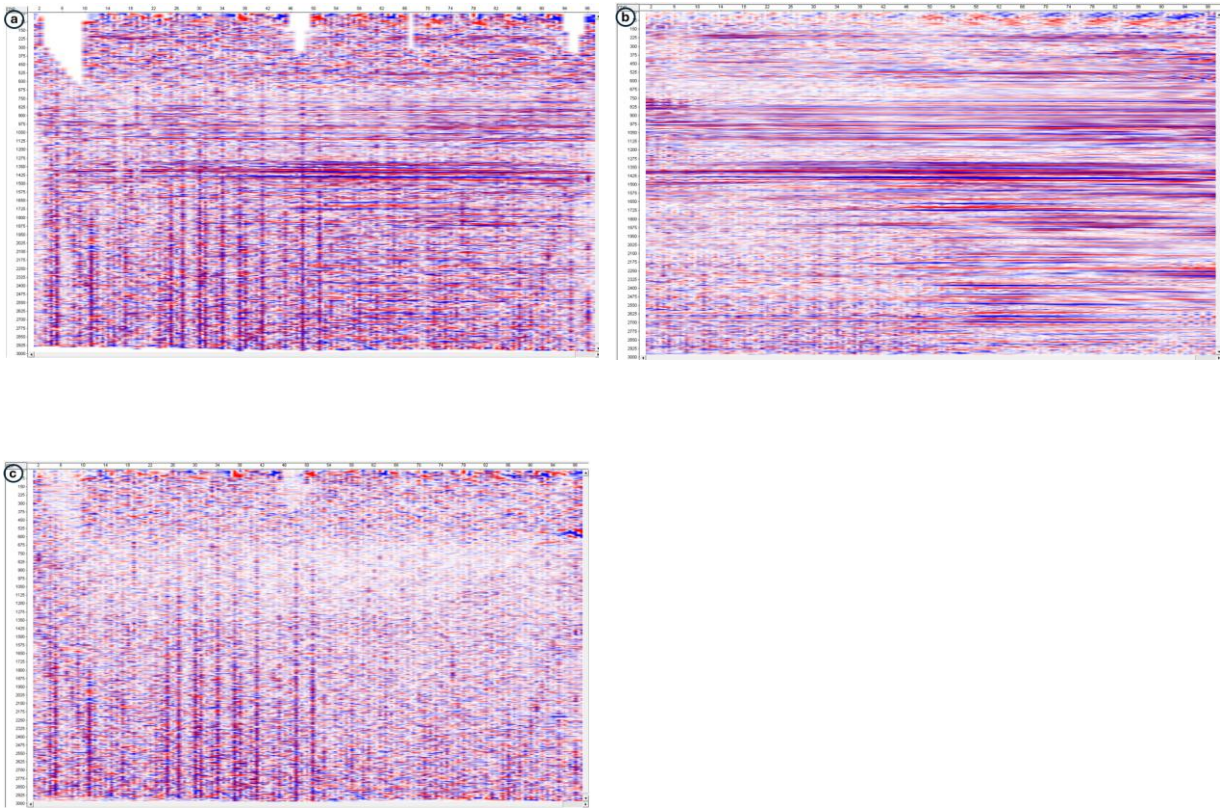


Figure 2: Panel (a) shows the input seismic data from WCSB contaminated with both Gaussian and non-Gaussian noise. Panel (b) presents the filtered seismic section obtained by applying the proposed algorithm. Panel (c) depicts the corresponding noise removed during the filtering process.

References

Chen, K. and Sacchi, M., 2015, Robust reduced-rank filtering for erratic seismic noise attenuation. Geophysics (80), V1-V11.

Golub, G.H. and Van Loan, C.F., 2013, *Matrix Computations. Fourth edition, Johns Hopkins University Press.*